

The physics of space and time II: A reassessment of Einstein's 1905 special relativity paper

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Abstract

A detailed re-examination of the seminal paper on special relativity, taking into account recent work on the physical interpretation of the space-time Lorentz transformation as well as the modern understanding of classical electromagnetism as a certain limit of the fundamental underlying theory —quantum electrodynamics— is presented. Many errors both of physical principle and of a mathematical nature are uncovered. The ‘relativity of simultaneity’ and ‘length contraction’ effects predicted in the paper are shown to be the spurious consequences of misinterpretations of the second postulate and the Lorentz transformation, respectively. The derivation of the latter in the paper is shown to be flawed. In this case, and other instances, due to cancellation of mistakes, a correct result is obtained in a fallacious manner. Separate lists of the correct and incorrect predictions of the paper are given. Due to the unique and revolutionary nature of its epistemological approach (the use of ‘thought experiments’ and axiomatic derivations) and the experimentally verified predictions of time-dilatation and mass-energy equivalence the fundamental importance of the paper for the development of physics is little affected by the many mathematical errors and (to modern eyes) physical misconceptions that it contains.

PACS 03.30.+p

1 Introduction

The present paper is intended as a companion to another paper on space-time physics [1] by the present author where it is attempted to construct the theory of flat ¹ space-time on the basis of simpler and more evident axioms than hitherto. When this is done, it is seen that certain predictions of standard Special Relativity Theory (SRT) — in particular ‘relativity of simultaneity’ of two supposedly synchronised, spatially separated, clocks in the same inertial frame and the associated ‘length contraction’ effect, derived by Einstein as a consequence of the Lorentz Transformation, in the seminal 1905 paper on SRT [2], are spurious ². They result from confusion between arbitrary clock offsets, completely controlled by the experimenter, and physical time intervals observed in different frames, completely controlled by the Lorentz transformation. As well as in Ref. [1], how this come about is explained in the papers [4, 5, 7]. It is also explained, in the present paper, in the commentary on Einstein’s analysis of the thought experiments where such effects were first introduced.

The motivation for writing the present paper is quite clear. As it is claimed that some results of Einstein’s original 1905 paper, which have since been propagated unchanged through several generations of text books, as well as the pedagogical literature ³ during the last century, are wrong, it is mandatory to explain just where, and for what reason, these conclusions of Einstein are incorrect.

The corrections to the interpretation of SRT result also in many changes in text-book formulae of classical electromagnetism, all dating from well before the advent of SRT. These formulae, many of which occur in Einstein’s original SRT paper, are referred to below under the acronym ‘CEM’ for Classical Electro-Magnetism, to be distinguished from the recent classical electrodynamic theory [8], of the present author, which is fully consistent with both SRT and, in the appropriate limit, with Quantum Electro-Dynamics (QED). This theory is referred to below as ‘RCED’ for Relativistic Classical Electro-Dynamics. The salient points concerning RCED are summarised below in the present section, while some detailed differences with CEM will become evident to the reader in the discussion of the ‘Electrodynamical Part’ of Ref. [2]. These differences, as well as others, are also discussed in Refs. [9, 10, 11].

The century since Einstein’s original paper on special relativity was written has seen an enormous increase in the understanding of physics. Perhaps the qualitatively most important discovery of 20th Century physics, and the one with the most important practical ramifications was that matter (and anti-matter) can be created and destroyed. The realisation that this was possible followed directly from the concept of the equivalence of

¹That is, for cases in which all gravitational effects may be neglected.

²It is important to notice that the relativity of simultaneity effect discussed by Einstein in Ref. [2] and also a popular book. [3], although, as will be shown below in Section 4, spurious, is not the same as the ‘text book’ relativity of simultaneity effect derived from a misapplication of the space-time Lorentz transformation, described in Section 7 below. The source of the relativity of simultaneity effect of Refs. [2, 3] is more a consequence of a wrong understanding of the operational meaning of Einstein’s second postulate of SRT, than of that of the Lorentz transformation.

³Most of this has appeared in two journals: The American Journal of Physics and the European Journal of Physics.

mass and energy introduced in Ref. [2]. The matter that is created or destroyed consists of elementary particles, the detailed properties of which have been revealed by a fruitful symbiosis between experiment and theory in the domain of ‘High Energy Physics’ in the second half of the 20th Century. By a historical quirk, the theory that describes the creation and destruction of elementary particles is called ‘Quantum Field Theory’ (QFT) to distinguish it from ‘Quantum Mechanics’(QM) in which matter exists permanently.

Indeed, as formulated by Feynman, the original and most successful QFT, QED [12] born of the marriage of QM with SRT, and describing the creation and destruction of real or virtual photons and electrically charged elementary particles, has, as its fundamental concepts, not ‘fields’ but the particles (which are what exist) and probability amplitudes that describe the histories, in space time, of these existing particles. These are both minimal and sufficient concepts to obtain all predictions of QED.

The most revolutionary conceptual discovery of 20th Century physics, apart from the modification of the concept of time required by SRT, at the basis of relativistic kinematics, is that of QM and QFT. Our best current fundamental knowledge is that the world is described in a probabilistic fashion by these theories. The ‘classical’ physics of the 19th Century and earlier ones is given by certain limits of this quantum description, typically those where the number of elementary particles involved is very large or where the kinematical properties of objects are such that their corresponding mechanical action is much larger than Planck’s constant h .

The above preamble is relevant to the subsequent discussion of Ref. [2] because of the crucial role of CEM in Einstein’s formulation of SRT, in the paper with the title ‘On the Electrodynamics of Moving Bodies’. Much of the material in even modern text books on CEM is still presented in terms of 19th Century concepts, and, as already mentioned, many standard formulae of the subject pre-date the discovery of SRT. It will be found that several of the electrodynamic problems discussed by Einstein in [2] can be most easily (and always, unlike in some of Einstein’s analyses, correctly) analysed in the language of ‘CEM as a limit of QED’. i.e. by introducing the photon concept, invented in the same year in an earlier paper [13] by Einstein as the ‘light quantum’, but never used in the SRT paper.

In a series of related papers, the present author has explored two related topics:

- (i) What can knowledge of CEM teach about QM?
- (ii) What can knowledge of QED teach about CEM?

The question (i) was addressed in a paper in which the comparison by ‘inverse correspondence’ of a plane, monochromatic, electromagnetic wave with the equivalent mono-energetic, parallel, beam of real photons, provides a simple understanding of many key concepts of QM [14]. The question (ii) was considered for the particular case of inter-charge forces, mediated by the exchange of virtual photons, in the absence of real photon radiation, in Ref. [8].

The elementary QED process underlying, say the magnetic force between two current-carrying conductors, is Møller scattering : $e^-e^- \rightarrow e^-e^-$ It is then assumed that the

laws of physics do not change between the situation in which just two electrons scatter from each other, or when the conduction electrons in one wire interact with those in another. This can be considered as an application of Newton's third rule of reasoning in philosophy [15]:

‘The qualities of bodies, which admit neither intension nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments are to estimated as the universal properties of all bodies whatsoever.’

In the present case, the word ‘bodies’ in this statement should be replaced by ‘electrons’. Two electrons in free space, whose fundamental mutual interaction is described by the Møller scattering amplitude, are the same as, and have the same mutual interaction as, the conduction electrons in a wire. There is no reason to believe that this interaction is modified, in any way, when the interacting electrons are separated by macroscopic distances. At lowest order, the QED momentum-space invariant amplitude for Møller scattering involves the exchange of a single, space-like, virtual photon. Fourier transforming this amplitude, expressed in terms of momenta in the overall centre-of-mass (CM) system, into space time, demonstrates the instantaneous, not retarded, nature of the intercharge interaction [8]. This is also evident from the relativistic velocity formula $v = pc^2/E$ to be discussed below in Section 13. In the CM frame the energy, but not the momentum, of the virtual photon vanishes so that its velocity, v , is infinite. A recent experiment measuring the distance-dependence of magnetic induction has verified experimentally this prediction [16, 17].

If a force is transmitted by isotropic emission and subsequent absorption of particles, the inverse square law is a simple consequence of conservation of the number of transmitted particles between the processes of emission and absorption. For large, space-like, intervals Δs , the Lorentz-invariant Feynman propagator of the corresponding space-like virtual particle, of pole mass m , is $\simeq \exp[-im\Delta\tau/\hbar]$ [18] where $\Delta\tau \equiv -i\sqrt{(\Delta x)^2 - c^2(\Delta t)^2} = -i\Delta s$. Therefore, for space-like intervals with $\Delta x > \Delta t$, the propagator does not vanish as predicted by ‘causality’ (see Section 13 below) but is instead exponentially damped $\simeq \exp[-m\Delta s/\hbar]$. For photons (or any other massless particle) for which $m = 0$ this damping does not occur, so that conservation of the number of virtual particles predicts that the corresponding exchange force law is inverse square. In this way, both Coulomb's inverse square law and the instantaneous nature of the intercharge force are first-principle predictions of QED.

An explicit relativistic formula for the inter-charge force in RCED (Eqn(3.4) below) has been obtained in Ref. [8] from the three postulates: an instantaneous Coulomb Law force in the non-relativistic limit, special relativistic invariance and Hamilton's Principle. A Lorentz-invariant Lagrangian for two interacting charges is constructed from the space-time and velocity 4-vectors of the two charges by imposing consistency with the well-known Lagrangian for the two-body central-force problem in the non-relativistic limit [19]. A logical ‘concept-flow’ diagram of the derivation is shown in Fig.1. All essential physics is contained in the six upper boxes, surrounded by the dot-dashed line. It is obtained without the introduction of any ‘force’ or ‘field’ concept. However, as shown by the boxes outside the dot-dashed line, all the concepts and laws of CEM developed during the 19th Century are derived by simple mathematical substitutions, either in the invariant Lagrangian or

in the interchange force equation. It is interesting to compare Fig.1 with the concept-flow diagram of Fig.2 which shows the logical development of the ideas of CEM during the 19th Century. The theoretical starting points here are the phenomenological concepts of electric and magnetic fields, introduced by Faraday and Maxwell and operationally defined in terms of forces exerted on a test charge. These are the concepts in terms of which Einstein originally formulated SRT. Note the similarity between Fig.2 and the bottom half, below the dot-dashed line, of Fig.1. However all the ‘Empirical Laws’ and ‘Classical Theoretical Predictions’ of Fig.2, with the exception of ‘ $E M$ waves’, are included in the single box ‘Intercharge Force’ of Fig.1.

The RCED predictions of Ref. [8] for the electric field of a uniformly moving charge and its associated magnetic field, Eqns(9.19) and (9.20) below, do not agree with the pre-relativistic Heaviside formulae [20]. This has important consequences for the discussion of the formulae in the ‘Electrodynamical Part’ of Ref [2]. As will be discussed in Section 3 below, the Heaviside formula has been demonstrated to be erroneous by calculating the electromagnetic induction effect produced by a simple two-charge ‘magnet’ in different reference frames [10]. Also, in Ref. [11], it is shown that there is an important mathematical error in the derivation if the retarded Liénard-Wiechert [21] potentials from which the ‘present time’ Heaviside formula for the retarded fields may be obtained. That is, it is not only that the interchange force is instantaneous, not retarded, but also that the Heaviside formula does not even correctly describe a hypothetical retarded interchange force. Correct formulae for the retarded fields of a uniformly-moving charge in the ‘present time’ form, directly comparable with the Heaviside formula may be found in Ref. [22].

In Ref. [8] the simplest possible system –just two interacting charges was considered. However in the ‘Electrodynamic Part’ of Ref. [2], the electric and magnetic fields introduced exist *a priori* and nowhere is there any discussion of how they are produced. As already pointed out in Ref [11] this has important consequences both on the very nature, in a mathematical sense, of the electromagnetic fields, as well as their transformation laws and the status of their relativistic covariance in the presence of interactions. This important point is discussed in some detail in Section 9 below.

Also, in common with most authors of text books on CEM, Einstein makes no distinction between the ‘force fields’ which correspond to the ‘Intercharge Forces’ of Fig.1 and ‘Electric Forces’, ‘Magnetic Forces’ and ‘ $E M$ Induction’ of Fig.2 and, ‘radiation fields’, not considered Fig.1, and describing ‘ $E M$ Waves’ in Fig.2, using identical mathematical symbols to describe them both. Actually the operational meanings of the two types of fields are quite distinct [23, 8, 11]. This difference will be taken into account in the discussion of the various sections of the ‘Electrodynamic Part’ of Ref. [2].

Turning now to the ‘Kinematical Part’ of Ref. [2] it is clear that Einstein’s presentation of SRT results in a strong entanglement (in the common, not the quantum mechanical technical sense) of concepts from space-time physics and CEM. In Fig.3 is shown a concept flow diagram of the Larmor-Lorentz [24, 25]⁴ and Einstein derivations of the LT and of

⁴It is difficult to understand why Larmor’s work, published in 1900, was not cited in Lorentz’ 1904 paper where identical transformation equations were given. The appellation ‘Lorentz Transformation’ is due to Poincaré [26]. See Ref. [27] for a discussion of priority issues related to the discovery of the Lorentz Transformation.

Voigt's derivation of his earlier, and closely related⁵ transformation [28]. Of particular importance for Einstein's formulation of SRT is the light-signal synchronisation procedure for spatially separated clocks, that assumes spatial isotropy of the speed of light. Relaxing the latter postulate leads to the 'conventionality' of clock synchronisation by this method, a subject on which a vast literature exists [29].

However, applying Occam's razor to the derivation of the LT, in a similar way to that in which it was applied to CEM in Ref. [8] to obtain RCED, is there, in fact, any necessary connection between SRT and CEM? It was realised very early that the answer to this question is 'No'. Of particular importance in this context is the 1910 paper of Ignatowsky [30] in which the necessary existence of a limiting relative velocity, V , between any two inertial frames was pointed out. Indeed, a very extensive literature on 'lightless' derivations of the LT exists. See Ref. [31] for citations of work prior to 1968, and the paper published some years ago by the present author [32] giving later citations, and in which the one-dimensional LT was derived from just two postulates, Single-Valuedness(SV) and the Measurement Reciprocity Postulate (MRP):

SV: Each Lorentz transformation equation must be a single-valued function of its arguments.

MRP: Reciprocal space-time measurements of similar rulers and clocks at rest in two different inertial frames: S, S' by observers at rest in S', S , respectively, give identical results.

Another, much shorter, derivation can be found in Ref. [33] based on the postulates SV and Space-Time Exchange Invariance (STEI):

STEI: The equations describing physical laws are invariant with respect to the exchange of space and time coordinates, or more generally, with respect to the exchange of spatial and temporal components of 4-vectors.

In order to derive the full three-dimensional LT the additional postulate of spatial isotropy is required in both cases. In the derivation from SV and the MRP the limiting velocity V appears naturally in the course of the derivation, as was the case in Ref. [30]. In the one based on SV and STEI the universal parameter, V , with the dimensions of velocity, must be introduced, for dimensional reasons, in order to define the space-time exchange operation, that requires unidimensional space and time coordinates. The concept flow diagram of the LT derivations of Refs. [32, 33] in Fig.4 may be compared with that in Fig.3. No appeal is made to CEM, or any other dynamical theory. As discussed below, Einstein's second postulate of the constancy of the speed of light is *derived* (see Fig.4) from relativistic kinematics and the massless nature of the photon [32]. The LT derivations of Refs. [32, 33] are recalled in the Appendix of Ref. [1].

In Ref. [1], several clock synchronisation procedures, not making any use of light signals are introduced. The crucial point (as clearly stated by Einstein himself, although never

⁵The Voigt transformation is obtained from the LT by multiplying the right sides of all the equations of the latter by the factor $\sqrt{1 - (v/c)^2}$.

correctly applied by him, or by anyone else) is that, as will be shown in the following section, the space-time LT describes, fundamentally, how times recorded by clocks in uniform motion appear to a ‘stationary’ observer. If there are several such synchronised clocks in the same moving inertial frame, at different spatial positions, suitable constants must be added, for each such clock, to the right sides of the equations of the conventional space time LT. The latter describes correctly only a synchronised clock at the origin of the moving frame. Until now, this has not been done, resulting, as described previously in Refs [1, 4, 5, 7], in the spurious ‘relativity of simultaneity’ (RS) and ‘length contraction’ (LC) effects of conventional SRT.

The following section contains, for the reader’s convenience, a brief summary of the correct physical interpretation of the space-time LT given in Refs. [1, 4, 5, 7]. In the subsequent sections the spurious arguments by which Einstein arrived at the predictions of RS and LC are examined in detail. Sections 3-13 contain commentaries on §1-§10 of Ref. [2]. Section 14 contains a summary that includes a list of Einstein’s mistakes, and the concluding Section 15 discusses the legacy of the 1905 special relativity paper in the light of both the findings of the present reassessment and its historical context.

Since Einstein did not number the equations of Ref. [2] (there are $\simeq 140$ in the paper) a complete numbered list is presented, section-by-section, in the Appendix. They are numbered with an ‘E’ in correspondence with the numbered sections of Ref. [2]. e.g., (E10.2) is the second equation of §10. Although all the equations in the Appendix are fair copies of those in the English translation of Ref. [2], in places the notation has been changed to be more in conformity with modern usage. For example γ for $1/\sqrt{1 - (v/c)^2}$ and β for v/c , as well as 3-vector notation for components of electric and magnetic fields. Also, depending on what is most convenient, in some places Einstein’s original notation for space-time coordinates, and in others that of Section 2 below, will be employed. The only ambiguous symbol is τ which stands, in Einstein’s notation, for the apparent time of clocks in the ‘moving’ frame S’ as viewed from the ‘stationary’ frame S, and in the notation of Section 2 for the proper time of clocks at rest in S as viewed in this frame. Forewarned of this, the reader should not be too confused by the use of the two different nomenclatures. For clarity, the most important E-numbered equations are repeated in the text at the appropriate place. All other equations are numbered according to the section of the present paper in which they occur.

2 Description of a clock in uniform motion by the Lorentz or Galilean transformation

The space-time LT connects observations of the same space time event in different inertial frames. If the frame S’ moves with speed v along the x -axis of the frame S, and if the Cartesian axes of the frames are parallel, the LT is written as:

$$x' = \gamma[x - vt] \tag{2.1}$$

$$t' = \gamma[t - \frac{\beta x}{c}] \tag{2.2}$$

$$y' = y \quad (2.3)$$

$$z' = z \quad (2.4)$$

where $\beta \equiv v/c$, $\gamma \equiv 1/\sqrt{1-\beta^2}$ and c is the speed of light in vacuum. The space-time coordinates of the transformed events are (x,y,z,t) in S and (x',y',z',t') in S' . In this section, for simplicity, only events lying on a common x - x' axis are considered.

To give an operational meaning to the time coordinates, it is necessary to introduce clocks; say C at rest in S and C' at rest in S' . The time symbols in the LT are then identified with the readings of these clocks. The LT can then describe two different experiments:

(i) C' (in motion) is viewed from S

(ii) C (in motion) is viewed from S'

The appropriate x - t LT equations to describe these two possibilities are, for experiment (i):

$$x'(C') = \gamma[x(C') - v\tau] = 0 \quad (2.5)$$

$$t' = \gamma[\tau - \frac{\beta x(C')}{c}] \quad (2.6)$$

and for experiment (ii):

$$x(C) = \gamma[x'(C) + v\tau'] = 0 \quad (2.7)$$

$$t = \gamma[\tau' + \frac{\beta x'(C)}{c}] \quad (2.8)$$

Note that (2.7) and (2.8) are not simply the inverse transformation of (2.5) and (2.6) but four different time symbols appear in the equations with different operational meanings:

τ : proper time recorded by C in S

τ' : proper time recorded by C' in S'

t : apparent time of C as viewed from S'

t' : apparent time of C' as viewed from S

Also four different space coordinates: $x(C)$, $x(C')$, $x'(C)$ and $x'(C')$ occur. Inspection of (2.5) and (2.6) shows that $t' = \tau = 0$ when $x = 0$. This means, operationally, that the clocks C and C' are synchronised (set to zero) at the instant that the clock C' at $x' = 0$ is at the x -origin in S . If it is now desired to synchronise, in a similar manner, the clocks C and C' when C' is at $x' = L'$, and its x coordinate in S is L , the equations (2.5) and (2.6) require modification. As correctly pointed out by Einstein [34]:

If no assumption whatever be made as to the initial position of the moving system and as to the zero point of τ , (t' in the notation used above) an additive constant is to be placed on the right side of each of these (the LT (2.1)-(2.4)) equations.

A suitable modification of (2.5) and (2.6) is:

$$x'(C') = \gamma[x(C') - v(\tau - \tau_0)] \quad (2.9)$$

$$t' - t'_0 = \gamma[\tau - \tau_0 - \frac{\beta x(C')}{c}] \quad (2.10)$$

where the constants τ_0 and t'_0 are to be chosen in such a way that $t' = \tau = 0$ when $x(C') = L$, $x'(C') = L'$. This requires that:

$$L' = \gamma[L + v\tau_0] \quad (2.11)$$

$$t'_0(L) = \gamma[\tau_0 + \frac{\beta L}{c}] \quad (2.12)$$

Solving these equations gives:

$$\tau_0 = \frac{1}{v}(\frac{L'}{\gamma} - L) \quad (2.13)$$

$$t'_0(L) = \frac{1}{v}(L' - \frac{L}{\gamma}) \quad (2.14)$$

Substituting these parameters in (2.9) and (2.10) gives:

$$x'(C') - L' = \gamma[x(C') - L - v\tau] \quad (2.15)$$

$$t' = \gamma[\tau - \frac{\beta}{c}(x(C') - L)] \quad (2.16)$$

Since $L \equiv x(C', \tau = 0)$ and $L' \equiv x(C', t')$, for all values of t' , are constants, independent of v , depending only on the choice of spatial coordinate systems in S and S' respectively, Eqn(2.15) holds for all values of v . In particular, when $v = 0$, so that $S \rightarrow S'$, $\gamma \rightarrow 1$ and $x \rightarrow x'$, it gives:

$$x'(C') - L' = x'(C') - L \quad (2.17)$$

or

$$L' = L \quad (2.18)$$

Since $x'(C') = L'$, (2.15) gives, for the equation of motion of C' in S:

$$x(C') = L + v\tau \quad (2.19)$$

Substituting (2.19) into (2.16) and recalling the definition of γ gives the time dilatation (TD) formula:

$$\tau = \gamma t' \quad (2.20)$$

Equations (2.19) and (2.20) give the complete space-time description of the clock C' at $x' = L$, as viewed from S.

Note that (2.19) is also valid for the Galilean transformation (i.e. the limit of the LT when $c \rightarrow \infty$), whereas the latter gives, instead of (2.20), $\tau = t' \equiv T$ where T is a universal (Newtonian) time.

Repeating the above chain of arguments for the experiment (ii), where the clock C is viewed from the frame S', gives the equations describing this (reciprocal) experiment:

$$x'(C) = L - vt' \quad (2.21)$$

$$\tau' = \gamma t \quad (2.22)$$

Introducing now two clocks in S', C'_A at $x' = 0$ and C'_B at $x' = L$ (2.19) and (2.20) give:

$$x(C'_A) = v\tau \quad (2.23)$$

$$x(C'_B) = L + v\tau \quad (2.24)$$

$$\tau = \gamma t'(C'_A) \quad (2.25)$$

$$\tau = \gamma t'(C'_B) \quad (2.26)$$

Eqns(2.23) and (2.24) give:

$$x(C'_B) - x(C'_A) = L = x'(C'_B) - x'(C'_A) \quad (2.27)$$

so there is no ‘length contraction’ (LC) effect, while (2.25) and (2.26) give:

$$t'(C'_A) = t'(C'_B) \equiv t' \quad (2.28)$$

where t' is the apparent time, as viewed in S, of *all* synchronised clocks in S'. There is therefore no ‘relativity of simultaneity’ effect in this case. How these effects arise from misuse of the LT (2.1)-(2.4) has been previously explained [1, 4, 5, 7] and will become clear in the following sections of the present paper where Einstein’s predictions of these effects are critically examined.

It will become apparent in the following discussion of Ref. [2] that Einstein did not take into account the existence of the four operationally distinct time symbols appearing in the LTs (2.5)-(2.8). All discussions are based on (2.1) and (2.2) where t is identified with the proper time in S and t' with the proper time in S'. This is equivalent to making the substitutions $t \rightarrow \tau$ in Eqn(2.8) and $t' \rightarrow \tau'$ in Eqn(2.6). That this is a logical absurdity becomes evident on making the same substitutions in (2.20) and (2.22) leading to the equations:

$$\tau = \gamma \tau' \quad (2.29)$$

$$\tau' = \gamma \tau \quad (2.30)$$

These equations require that $\gamma^2 = 1$ or $v = 0$, in contradiction to the initial hypothesis that the the velocity, v , of S' relative to S, is non-vanishing.

In some applications it is convenient to introduce an alternative notation for the apparent time intervals t' and t :

$$t' \equiv \tau'(C') \quad (2.31)$$

$$t \equiv \tau(C) \quad (2.32)$$

Here, $\tau'(C')$, for example, is a proper time interval recorded by the clock C', at rest in S', and observed in this frame. This is to be contrasted with τ' in Eqn(2.22) that is, instead, the proper time interval in S' corresponding to the observation of the time t of a clock at rest in S, but observed from S'. Use of the symbols τ , τ' , t and t' has the advantage that they all represent direct observations of time intervals of clocks, either at rest (τ , τ'), or

in motion (t, t'). Since these times are the same for all synchronised clocks in the frames, no clock labels are necessary. If (2.31) and (2.32) are used, all symbols specify proper times but additional clock labels are required to distinguish, for example, τ and $\tau(C)$.

An important application of (2.31) occurs in particle physics where the clock C' is identified with an unstable particle P , and $\tau'(C' \equiv P)$ with its proper decay time. In this case (2.20) and (2.29) give:

$$\tau'(C' \equiv P) = \frac{\tau}{\gamma} \quad (2.33)$$

where τ is the observed decay lifetime of the particle in the laboratory system and the time dilatation factor, γ , is derived from the measured speed, v , of the particle in the laboratory system. In this case the proper time interval $\tau'(C' \equiv P)$ is not directly observed, but deduced from the TD relation (2.33) and the observed laboratory lifetime $\tau(P)$.

Another application of Eqn(2.31) occurs in the discussion of relativistic velocity addition in Section 8 below.

3 Einstein's Introduction: The physics of electromagnetic induction in different reference frames

In the introductory section of Ref. [2] Einstein discusses the phenomenon of electromagnetic induction in two reference frames: the first in which a magnet is in motion and a test charge is at rest, and the second in which the magnet is at rest and the test charge in motion. For the present discussion, the test charge can be conveniently considered to be at rest in the frame S' , while S is the magnet rest frame. Einstein states that in the first case an electric field acts on the test charge, but did not attempt to calculate this field. As shown below, if he had, he might have obtained a very surprising result! In the case that the magnet is at rest and the test charge in motion, the latter moves in a inhomogeneous but static magnetic field and is subjected a transverse Lorentz force which, in this case, explains the induction effect. Einstein states that, in this second case, 'no electric field occurs in the neighbourhood of the magnet'. Later in Ref. [2], after discussing the transformation laws of electric and magnetic fields, it is pointed out that the magnetic (Lorentz) force in one frame (that in which the test charge is in motion) becomes an electric force in the frame in which the test charge is at rest, as a direct consequence of the transformation laws of electric and magnetic fields. Thus the magnetic and electric forces are just the descriptions in different frames of the force on the test charge responsible for electromagnetic induction. The two separate phenomena discussed by Einstein are thus unified in special relativity by the transformation laws of electric and magnetic fields. However, explicit calculation of the force on the test charge in the two cases, using standard formulae of CEM, will now be shown to lead to a less satisfactory conclusion.

Consider a test charge, q , near an elementary 'magnet' consisting of two charges of magnitude Q situated, at some instant, at the points U and D (See Fig.5). The points U

and D lie along the y axis equidistant from the origin, whereas the charge q lies in the x - y plane displaced, by a small distance, from the x -axis. In Fig.5a the charges at U and D move parallel to the positive and negative z axes, respectively, with speed u . The velocity vectors of the charges are \vec{u}_U and \vec{u}_D , where $\vec{u}_U = -\vec{u}_D$ and $|\vec{u}_U| = |\vec{u}_D| = u$. The test charge moves with speed v in the positive x -direction. An imaginary rectangular contour, abcd, parallel to the y - z plane moves together with the test charge. Fig.5a corresponds to the second case considered by Einstein —stationary magnet and test charge in motion. Because of the displacement of the test charge from the symmetry axis, Ox , there is a net magnetic field component parallel to the y -axis. The corresponding Lorentz force is in the direction of the negative z -axis and explains the induction effect in the rest frame of the magnet.

In Fig.5b is shown the corresponding configuration in the frame S' where the test charge is at rest and the magnet moves with speed v along the negative x' -axis. The velocity vectors \vec{u}'_U and \vec{u}'_D of the charges at U and D are now at an angle α to the z' -axis where $\sin \alpha = \beta/\beta_u = \gamma\beta/\sqrt{\beta_u^2 + \gamma^2\beta^2}$ and $\beta_u = u/c$. Since the test charge is at rest there is no Lorentz force, so, as pointed out by Einstein, electromagnetic induction can be produced in this frame only by an electric field. The electric field vectors \vec{E}_U and \vec{E}_D , at the point where the plane abcd cuts the x' -axis, are shown as predicted by the Heaviside formula [20] for the electric field of a uniformly moving charge:

$$\vec{E}(\text{CEM}) = \frac{Q\vec{r}}{r^3\gamma_u^2(1 - \beta_u^2 \sin^2 \psi')^{\frac{3}{2}}} \quad (3.1)$$

The radius vector \vec{r} is that from the source charge to the field point (so the electric field is predicted to be radial) and ψ' is the angle between the radius vector and the source charge velocity vector \vec{u}' . Because of the radial character of the electric field, the vectors \vec{E}_U and \vec{E}_D associated with the source charges U and D, lie in the x' - y' plane at every point in this plane, including that at the position of the test charge. There is therefore no electric field in the z' -direction, and so, according to Eqn(3.1), no electromagnetic induction effect in the frame S' . The Heaviside Formula (3.1) is therefore inconsistent with SRT for the configuration of moving charges shown in Fig.5.

In the recent paper by the present author [8], mentioned in the Introduction, a different formula to (3.1) for the electric field of a moving charge was obtained:

$$\vec{E}(\text{RCED}) = \frac{Q}{r^2} \left[\frac{\hat{u}' \cos \psi'}{\gamma_u} + \hat{t}' \gamma_u \sin \psi' \right] \quad (3.2)$$

where \hat{u}' is a unit vector parallel to the source charge velocity and \hat{t}' is a unit vector perpendicular to \hat{u}' lying in the plane defined by \hat{u}' and the radius vector \vec{r} specifying the field point, on the same side as the field point. The electric field defined by (3.2) is not radial and calculation [10] in the frame S' shows agreement at $O(\beta^2)$ ⁶ with the induction effect calculated in the frame S , either by explicitly evaluating the Lorentz force in this frame, or by use of the integral form of the Faraday-Lenz Law in either S or S' .

It is interesting to note, in connection with the previous remark, that the problem of the apparent inconsistency of the description of induction in different frames, posed by

⁶Due to the lack of covariance of the longitudinal electric field (see Section 6 below) the induction force differs between the frames S and S' by terms of $O(\beta^4)$ and higher.

Einstein, does not arise if the integral form of the Faraday-Lenz law:

$$-\frac{d\phi}{dt} = \int \vec{E} \cdot d\vec{S} \quad (3.3)$$

is used to solve the problem. In Fig.5a, the flux ϕ of magnetic field threading the contour abcd changes because the contour moves in a static, but non-uniform, magnetic field, whereas in Fig.5b, the contour is stationary but ϕ changes because of the motion of the magnet. Typically, in text books, it is pointed out that the Faraday-Lenz law can be used to derive electromagnetic induction in either the frame S or S', whereas a fundamental dynamical understanding (the Lorentz force) is available only in the frame S. This situation provoked the following statement by Feynman [35]:

We know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of *two different phenomena*. Usually such a beautiful generalisation is found to stem from a single *deep underlying principle*. Nevertheless, in this case there does not appear any such profound implication. We have to understand the rule as the combined effects of two quite separate phenomena. (Italics in the original).

The ‘separate phenomena’ referred to are the Lorentz force law and the Faraday-Lenz law (3.3). In fact a completely unified description of electromagnetic induction as well all the other mechanical effects⁷ of classical electromagnetism is provided by the following formula [8] containing no fields, that gives the force on a test charge due to a source charge in arbitrary motion:

$$\frac{d\vec{p}}{dt} = \frac{q}{c} \left[\frac{j^0 \vec{r} + \vec{\beta} \times (\vec{j} \times \vec{r})}{r^3} - \frac{1}{cr} \frac{d\vec{j}}{dt} - \vec{j} \frac{(\vec{r} \cdot \vec{\beta}_u)}{r^3} \right] \quad (3.4)$$

Here q and $\vec{\beta}c$ are the charge and velocity of the test charge of Newtonian mass m and relativistic momentum $\vec{p} = \gamma\vec{\beta}mc$, and $j \equiv Q(\gamma_u c, \gamma_u \vec{u})$ is the 4-vector current of the source charge. The unified description of electromagnetic induction provided by Eqn(3.4) would no doubt have pleased both Einstein and Feynman. As discussed in Ref. [8] ‘the single deep underlying principle’ underlying Eqn(3.4) is the Feynman path integral formulation of quantum mechanics, applied to space-like virtual photon exchange processes, in the classical ($\hbar \rightarrow 0$) limit. This is the fundamental ‘constructive physics’ underlying both the Coulomb force law and Hamilton’s Principle, from which Eqn(3.4) is derived.

It has been pointed out in Ref. [11] that there is an important mathematical error in the derivation of the Liénard-Wiechert retarded potentials [21] from which the ‘present time’ Heaviside formula (3.1) is obtained. When this error is corrected then, up to an overall multiplicative factor, the formula (3.2) (but with a retarded time argument) is recovered.

After the discussion of electromagnetic induction, Einstein introduces the postulates on which his analysis of space-time physics is based: The Relativity Principle and the

⁷These include the relativistic generalisations of Coulomb’s Law and the Biot and Savart Law as well as the Lorentz force equation.

postulate that the speed of light does not depend upon that of the emitting body. Here, and later when explicitly defining this postulate it is not stated that the speed of light as measured in *any* inertial frame is constant. Einstein refers only to the ‘stationary’ frame S. Also, in the introduction, the Relativity Principle is applied only to the description of electromagnetic and optical phenomena in different reference frames. In accordance with the title, it is stated that the aim of the paper is to provide ‘a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell’s theory of stationary bodies’. In this theory the ‘luminiferous aether’, such an important concept of 19th Century electromagnetic theory, will become superfluous.

In the final paragraph of the introduction, Einstein stresses the importance of actual measurements of length intervals with rulers, and time intervals with clocks, in constructing the theory. This is certainly a very important remark. However, as shown in Ref. [1], the consideration of electromagnetic and optical phenomena, as in Einstein’s paper, is inessential to establish the foundations of space time physics in flat space, which are the space-time LT and its correct physical interpretation. The last sentence of the introduction is:

Insufficient consideration of these circumstances (i.e. actual, experimental, space and time measurements) lies at the root of the difficulties which the electrodynamics of moving bodies at present encounters.

As will be demonstrated below, this sentence remains valid one century after the work presented in Ref. [2] if the words ‘the physics of space time’ are substituted for ‘the electrodynamics of moving bodies’ in the sentence just quoted.

4 §1. Definition of Simultaneity

At the beginning of this section, Einstein stresses again the importance of actual clock readings and the concept of simultaneous events in establishing the meaning of physical time:

We have to take into account that all our judgements in which time plays a part are always judgements of *simultaneous events*. If, for instance, I say ‘‘That train arrives here at 7 o’clock’’, I mean something like this: ‘‘The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events’’.

What Einstein is stating here is that any actual time measurement (and therefore any measurement of a time interval) is built up from pointer-mark coincidences, which constitute the irreducible experimental raw data, and the concept of simultaneity. In the above example two, simultaneous, pointer-mark coincidences (between the hand of the clock and a mark on its dial, and between the front of the train and the position on a platform of Bern railway station where Einstein happened to be standing) are involved. In a similar way the raw experimental data of data of any spatial measurement involves, or is equivalent to, the spatial coincidence of a pointer specified by the geometry of some physical object and a mark on a ruler. A mathematically rigorous calculus of such pointer-

mark coincidences in constructing measurement of space and time intervals is developed by the present author in Ref. [1].

Einstein then addresses the problem of establishing the simultaneity of spatially separated events in a given reference frame, and proposes the light-signal clock synchronisation procedure of Eqns(E1.1) and (E1.2). This procedure immediately couples the concept of simultaneity with the physics of light propagation. In particular, Einstein's definition of simultaneity (or equivalently of clock synchronisation) assumes spatial isotropy of the speed of light. However, as shown in Ref. [1] it is perfectly possible to synchronise spatially separated clocks without using light signals, and so independently of the assumption of light-speed isotropy. Two such procedures: 'Pointer Transport' and 'Length Transport' are described in Ref. [1]. Pointer Transport can be used to synchronise an arbitrary number of spatially-separated clocks in the same reference frame, whereas Length Transport synchronises four clocks, two in each of two different inertial frames. Given the isotropy of the speed of light, Einstein's light-signal synchronisation procedure is, of course, a perfectly valid one. However applications of this idea to the discussion of the relativity of length and time intervals and the derivation of the LT in subsequent sections of Ref. [2] lead to serious errors in the interpretation of the physical meaning of Einstein's second postulate.

5 §2. On the Relativity of Lengths and Times

At the begining of this section Einstein restates his initial postulates⁸ of special relativity:

1. The laws by which the states of physical systems undergo change are not affected whether these changes of state be referred to one or the other of two systems of coordinates in uniform translatory motion.

2. Any ray of light moves in the "stationary" system of coordinates with the determined velocity, c , whether the ray be emitted by a stationary or a moving body. Hence:

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}} = c \quad (\text{E2.1})$$

where 'time interval' is to be taken in the sense of the definition in §1⁹.

Postulate 1 applies equally in classical (Newtonian) mechanics and was well known to both Galileo and to Newton. The postulate 2 was introduced, for the first time, by Einstein and is the basis of his formulation SRT. The above statement of this postulate refers only to Einstein's 'stationary' coordinate system (S in the notation of the present paper). It

⁸As will be seen, others are introduced, either explicitly or tacitly during the derivation of the LT. See Fig.3.

⁹That is, as defined by appropriate clocks synchronised by the light signal procedure defined in §1 of Ref. [2].

is only when the relation (E2.1) is further assumed to hold in an *arbitrary* inertial frame that its highly non-intuitive nature becomes evident. The latter was discussed in some detail in Einstein's later popular book on relativity [3]. However, it is quite unnecessary to introduce Eqn(E2.1) as a stand-alone postulate. Given that light consists of massless particles, photons (for the discovery of which, in another paper [13], published earlier in 1905, Einstein was awarded the Nobel Prize) Postulate 2 follows directly from relativistic kinematics [32]. The definitions of the relativistic energy and momentum of a physical object of Newtonian mass m :

$$E \equiv \gamma mc^2 \quad (5.1)$$

$$\vec{p} \equiv \gamma m\vec{v} \quad (5.2)$$

and of the relativistic parameter γ give:

$$v = \frac{pc^2}{E} = \frac{pc^2}{(m^2c^4 + p^2c^2)^{\frac{1}{2}}} \quad (5.3)$$

Hence, any massless particle has the velocity $v = c$ in any inertial reference frame. This is exactly the relation (E2.1), not only in the 'stationary', but in *any* inertial reference frame. The kinematical relations (5.1)-(5.3) are derived and discussed for the case of a object subject to electromagnetic forces in Section 13 below.

After describing carefully and correctly how the length of a measuring rod is defined both in the frame S' , in which it is rest, and that, S , in which it is in uniform motion, Einstein introduces a thought experiment based on the light signal clock synchronisation procedure (LSCSP) introduced in the previous section. This experiment is purported to give a direct demonstration of 'relativity of simultaneity' (RS):

We imagine further that at the two ends A and B of the rod, clocks are placed which synchronise with clocks of the stationary system, that is to say that their indications correpond at any instant to the "time in the stationary system" at the places that the happen to be. These clocks are therefore "synchronous in the stationary system.

We imagine further that with each clock there is a moving observer and that these observers apply to both clocks the criterion established in §1 for the synchronisation of two clocks.

The clocks are situated at the ends of the rod and so are at rest in S' . It follows from the TD formula (2.18) that these clocks must be running faster than a clock at rest in S by the factor γ in order to remain synchronous with it. It is clear however from the discussion of Section 2 above, in particular Eqn(2.26), that these clocks must also be synchronous for an observer in S' . Einstein states just the contrary. Why? In fact Einstein does not discuss the LSCSP in the frame S' , as performed by the 'moving observers', but instead considers a series of events defined in, and observed from, the frame S :

Let a ray of light depart from A at the time t_A where 'time' here denotes 'time of the stationary system' and also 'position of the hands of the moving clock situated at the place under discussion'. Let the ray be reflected at B at time t_B and reach A again at the time t'_A . Taking into consideration the principle of the constancy of the speed of light we find that

Event	$\tau' = \tau'_A/\gamma = \tau'_B/\gamma$	x'	τ	x
Light signal at A	0	0	0	0
Reflection from M'	L/c	L	$\gamma L/c$	$L(1 + \gamma\beta)$
Reflection from M	$L/[\gamma(c - v)]$	$L[2 - \gamma(1 + \beta)]$	$L/(c - v)$	$cL/(c - v)$
Return to A in S'	$2L/c$	0	$2\gamma L/c$	$2\gamma\beta L$
Return to A in S	$2\gamma L/c$	$-2(\gamma - 1)L$	$2\gamma^2 L/c$	$2\gamma^2\beta L$

Table 1: *Times and positions in S (τ and x) and in S' (τ' and x') for the events shown in Fig.6*

$$t_B - t_A = \frac{r_{AB}}{c - v} \quad (\text{E2.2})$$

$$t'_A - t_B = \frac{r_{AB}}{c + v} \quad (\text{E2.3})$$

where r_{AB} denotes the length of the rod --measured in the stationary system.

All of the above times are specified in the frame S, whereas the LSCSP for clocks at rest in S', as specified according the the prescription of §1 is based on the equations

$$t_B - t_A = t'_A - t_B \quad (\text{E1.1})$$

$$\frac{2AB}{t'_A - t_A} = c \quad (\text{E1.2})$$

where all times are those of clocks at rest in the frame S', the proper frame of the rod, and AB is the length of the rod in this frame. Einstein's conclusion:

Observers moving with the moving rod would thus find that the two clocks were not synchronous while observers in the stationary system would declare the clocks to be synchronous.

is false. This is because the events observed in S appearing in Eqns(E2.2) and (E2.3) are not the Lorentz-transformed LSCSP events defined in the frame S'. These latter events as observed either in S' or S, as well as the events in S considered by Einstein are shown in Table 1 and Fig.6.

It is assumed that, in addition to the mirror, M', at B, there are also light detectors (not shown) at A and B connected to the lamps L_A and L_B which flash when photons are detected near A and B. For example, in Fig.6a, L_A flashes when the light signal is close to A. The flashes of the lamps are visible to observers in both S and S' and so establish the correspondence between the same event seen in the two frames. Events as observed in the frame S' are shown on the left side of Fig.6, the same events, as observed in S, on the right side. Figs.6a, 6b and 6d show the events of the light signal synchronisation procedure as performed in the frame S', while Figs.6a, 6c and 6e show the events considered by Einstein in the frame S. In the frame S, the light signal is reflected from the mirror M that is in spatial coincidence with M' at the instant of reflection in S (Fig.6c, right).

Inspection of Table 1 and Fig.6 shows that there is no correlation between the events introduced by Einstein in the frame S, with times calculated according to the relations

(E2.2) and (E2.3) and the clock synchronisation events in the frame S' . The times indicated in Fig.6 are the proper times, as recorded by identical clocks, in these frames. The clocks at A and B introduced by Einstein, that are synchronous with a clock in S , record times: $\tau'_A = \tau'_B = \gamma\tau'$ where τ' is the proper time in S' as recorded by a clock identical to the one in S . Using Einstein's notation of (E1.1) for the times of events in S' (that is, setting, here, $\tau' = t$) the events shown in Table 1 and Fig.6 give:

$$t_A = 0 = \tau'_A \quad (5.4)$$

$$t_B = L/c = \tau'_B/\gamma \quad (5.5)$$

$$t'_A = 2L/c = (\tau'_A)'/\gamma \quad (5.6)$$

These equations give:

$$t_B - t_A = t'_A - t_B = L/c \quad (5.7)$$

and

$$\tau'_B - \tau'_A = (\tau'_A)' - \tau'_B = \gamma L/c \quad (5.8)$$

The last equation shows that the clocks at the ends of the rod that are synchronous with the clock in S will also be seen to be synchronous, according to Einstein's light signal procedure, by observers in the frame S' . This conclusion is contrary to Einstein's one.

Einstein's claim of non-simultaneity is based on (E2.2) and (E2.3), referring to the events in S shown in Figs.6c and 6e, and for which:

$$t_B - t_A \neq t'_A - t_B$$

Einstein incorrectly identifies these events with those of the synchronisation procedure in S' and so reaches the false conclusion that the clocks at A and B are not seen to be synchronous in S' .

It is clear however, from the very concept of Einstein's synchronisation procedure that both the photon source and the mirror must be at rest in some frame. Only then is the relation (E2.1) valid. The difference between $t_B - t_A$ and $t'_A - t_B$ for the events shown in Figs.6c and 6e has a trivial, and purely classical, explanation. The relative velocity of the light signal and the mirror M' , as observed in S , is $c - v$ before reflection and $c + v$ after reflection, whereas formula (E2.1), which is the basis of Einstein's synchronisation procedure requires the 'light path' to be fixed in some frame, not in motion, as is the case for equations (E2.2) and (E2.3). There is confusion here between the 'relative velocity' between a moving object and a light signal as observed in some frame, which can be greater or less than c , and the 'speed of light', defined by (E2.1), which is the constant c in any inertial frame.

Einstein mentions 'relativity of length' in the title of this section and makes the statement that a moving rod will be measured to have a different length from a stationary one. However no arguments are given to support this assertion, or mention made of the following §4, where Einstein claims to derive the 'length contraction' effect.

6 §3. Theory of the Transformation of Co-ordinates and Times from a Stationary System to another System in Uniform Motion of Translation Relative to the Former

In this section Einstein's notation for space-time coordinates is retained. However, as is now conventional, Einstein's frames K, k are denoted, as above, by S, S' respectively. An event as observed in S, S' then has space-time coordinates $(x, y, z, t), (\xi, \eta, \zeta, \tau)$ respectively.

Einstein's derivation of the LT is based on the LSCSP introduced in §1. However, the same false connection between events in the frames S and S' is assumed as in the discussion of relativity of simultaneity in §2. In spite of this, Einstein does succeed to derive the correct LT, at least the one appropriate to a synchronised clock situated at the origin of S' . Einstein assumes that the LT is linear in the space-time coordinates and seeks first to derive the time transformation equation. It would seem natural to assume the functional dependence $\tau = \tau(x, y, z, t)$ for this equation. Einstein however does not do this. It is assumed instead that $\tau = \tau(x', y, z, t)$ where:

$$x' \equiv x - vt = x(t = 0) = \text{constant} \quad (\text{E3.1})$$

even though the final result (E3.7) has the functional dependence $\tau = \tau(x, t)$. This, unlike the initial ansatz, does depend on x , (as it should), whereas the x' dependence (which must also be there) has been lost. How this comes about is explained below. Actually, for synchronised clocks, the time transformation (Eqn(2.18), in Einstein's notation):

$$\tau = t/\gamma \quad (6.1)$$

depends on neither x' nor x . It would seem however to be impossible to derive this equation by assuming simply $\tau = \tau(t)$ as an ansatz. Actually, (6.1) has been derived by combining (2.15) and (2.16). The latter equation has the functional dependence (in Einstein's notation) of $\tau = \tau(x', x, t)$ as it must have in order to correctly describe synchronised clocks with different values of x' . Note that in (2.15) and (2.16) Einstein's constant x' is L . Einstein's neglect of the essential x dependence of the LT in his initial ansatz is the first error in his derivation. Other, compensating, errors enable the correct transformation to be finally obtained.

Einstein next considers a light signal emitted from the origin of S' ; parallel to the ξ axis, that is reflected straight back by a mirror, and observed again at the origin of S' . This is just the sequence of events shown in Figs.6a, 6b, and 6d for which the space-time coordinates in the two frames are presented in the first, second and fourth rows of Table 1. However Einstein assumes, following the argument presented in §2, that the reflection event in S' is that shown in Fig.6c and the third row of Table 1, while the return-to-origin event in S is that in Fig.5e and the fifth row of Table 1.

The LSCSP relation (E3.2) that is the basis of Einstein's derivation of the LT will now be discussed, first inserting into it the correct times and positions in the frames S and S' from Table 1, and then, following Einstein's argument, with the functional dependence

$\tau = \tau(x', y, z, t)$, and assuming that the corresponding events in S are those in the third and fifth rows of Table 1.

Since the light signals have $y = \eta = z = \zeta = 0$, the corresponding time transformation equation has, in general, the functional dependence $\tau = \tau(x', x, t)$. The LSCSP relation in the frame S':

$$\frac{1}{2}[\tau_A + \tau'_A] = \tau_B \quad (6.2)$$

gives, noting that (see Fig.6) $x' = 0$ for the clock at A and $x' = L$ for the clock at B:

$$\frac{1}{2}[\tau(0, 0, 0) + \tau(0, 2\beta\gamma L, 2\gamma L/c)] = \tau(L, (1 + \gamma\beta)L, \gamma L/c) \quad (6.3)$$

Assuming linearity of the equations and partially differentiating (6.3) with respect to its arguments ¹⁰ gives:

$$\gamma\beta\frac{\partial\tau}{\partial x} + \frac{\gamma}{c}\frac{\partial\tau}{\partial t} = \frac{\partial\tau}{\partial x'} + (1 + \gamma\beta)\frac{\partial\tau}{\partial x} + \frac{\gamma}{c}\frac{\partial\tau}{\partial t} \quad (6.4)$$

or

$$\frac{\partial\tau}{\partial x} = -\frac{\partial\tau}{\partial x'} \quad (6.5)$$

consistent with Eqn(2.16) after the substitutions, to recover the notation of Section 2, $\tau \rightarrow t'$, $t \rightarrow \tau$ and $x' \rightarrow L$.

With the functional dependence $\tau = \tau(x', t)$ assumed by Einstein, and taking the corresponding events in S from the third and fifth rows of Table 1, the LSCSP relation (6.2) is written as:

$$\frac{1}{2}[\tau(0, 0) + \tau(0, \frac{x'}{c-v} + \frac{x'}{c+v})] = \tau(x', \frac{x'}{c-v}) \quad (E3.3)$$

Partial differentiation, assuming linearity gives:

$$\frac{\partial\tau}{\partial x'} + \frac{v}{c^2 - v^2}\frac{\partial\tau}{\partial x} = 0 \quad (E3.5)$$

In view of the linearity of the equations, (E3.5) is satisfied providing that:

$$\tau = a \left(t - \frac{v}{c^2 - v^2} x' \right) \quad (E3.7)$$

For the clock at A, where $x' = 0$, the time transformation is therefore

$$\tau_A = at_A \quad (6.6)$$

In agreement with (6.1). However $a = a(v)$ remains undetermined.

Since the clock at B has $x'_B = L$, it follows from (E3.7) that:

$$\tau_B = a \left(t_B - \frac{v}{c^2 - v^2} L \right) \quad (6.7)$$

¹⁰Einstein here makes the unnecessary assumption, in view of the postulated linearity of the equations that x' is infinitesimally small.

Eqn(E3.7) or (6.7), together with (6.6), already implies a ‘relativity of simultaneity’ effect. If $\tau_A = \tau_B$ then $t_A \neq t_B$ and vice versa. Comparing (E3.5) and (6.5) this can be seen to be the consequence of two errors in Einstein’s calculation:

- (i) Neglect of the essential x -dependence of τ .
- (ii) Substitution of incorrect corresponding events in the frame S in the LSCSP relation.

When the correct events in S are substituted into Eqn(6.2),(as in Eqn(6.4)), and the necessary x -dependence of the transformation is taken into account, the spurious relativity of simultaneity effect of Eqn(6.7) (the dependence of τ_B on L) is removed by the condition (6.5), equivalent to the substitutions $x' \rightarrow x' - L$ and $x \rightarrow x - L$ in the LT of Eqns(2.5) and (2.6) which describe a synchronised clock with $x' = L = 0$.

Einstein now considers a light signal emitted in the frame S’ at $\tau = 0$ along the ξ axis with the equation of motion:

$$\xi = c\tau \quad (\text{E3.8})$$

The time in S of the reflection event at M’ is then assumed (incorrectly, see Table 1 and Figs.6b,6c) to be

$$t_B = \frac{x'_B}{c - v} \quad (\text{E3.9})$$

Substituting this value of t_B in (E3.7) and using (E3.8) then gives the space-coordinate transformation equation:

$$\xi_B = a \frac{c^2}{c^2 - v^2} x'_B \quad (\text{E3.10})$$

Considering light signals moving along the η and ζ axes in S’ and assuming that the apparent speed of light in S is c leads to the relations:

$$\eta = a \frac{c}{\sqrt{c^2 - v^2}} y \equiv \phi(v)y \quad (\text{E3.16})$$

$$\zeta = a \frac{c}{\sqrt{c^2 - v^2}} z = \phi(v)z \quad (\text{E3.17})$$

Note that the y, z LT equations (2.3) and (2.4) ((E3.29) and (E3.30) in Einstein’s notation) together with the TD relation (6.1) predict, in accordance with Einstein’s assumption in deriving (E3.16) and (E3.17) that the apparent speed, in S, of light signals propagating perpendicular to the ξ axis in S’ is c . If r is the path length of the signal in S, then the apparent speed of light in S of a light signal propagating parallel to the η axis is:

$$c_{\eta}^{app} = \frac{r}{t} = \frac{\sqrt{y^2 + v^2 t^2}}{t} = \sqrt{\left(\frac{c\tau}{t}\right)^2 + v^2} = c \quad (\text{6.8})$$

where the time dilatation relation (6.1), $y = \eta = c\tau$ and the definition of γ have been used. This is to be contrasted with the apparent speed of light in S of signals moving

parallel to the ξ axis. The entries of Table 1 give the following apparent velocities for the photons following the paths AB and BA in S':

$$c_{\xi,AB}^{app} = c \left(\frac{1}{\gamma} + \beta \right) \quad (6.9)$$

$$c_{\xi,BA}^{app} = c \left(\frac{1}{\gamma} - \beta \right) \quad (6.10)$$

Substituting x' from Eqn(E3.1) in (E3.7) and (E3.10) gives, respectively:

$$\tau = \phi(v)\gamma(t - \frac{vx}{c^2}) \quad (E3.14)$$

$$\xi = \phi(v)\gamma(x - vt) \quad (E3.15)$$

where the subscript B is dropped. Einstein having now arrived at the space-time LT up to a multiplicative constant in (E3.14)-(E3.17), the use of the inverse transformation (tacitly assuming here the Reciprocity Principle [31, 32]) and spatial isotropy, to derive the relations:

$$\phi(v)\phi(-v) = 1 \quad (E3.25)$$

$$\phi(v) = \phi(-v) \quad (E3.26)$$

and hence $\phi(v) = \phi(-v) = 1$, is unexceptionable, giving finally the LT as:

$$\tau = \gamma(t - \frac{vx}{c^2}) \quad (E3.27)$$

$$\xi = \gamma(x - vt) \quad (E3.28)$$

$$\eta = y \quad (E3.29)$$

$$\zeta = z \quad (E3.30)$$

The correct LT (E3.27)-(E3.30) for a synchronised clock at A has now been derived by misapplication of Eqns(E3.14),(E3.15) to the clock at B, for which, lacking the required x' (or L) dependence of (2.15) and (2.16), they are not valid, and, in parallel, misinterpreting the meaning of Einstein's postulate 2, by assigning incorrect events in S in correspondence with the LSCSP events in S' (see Fig.6). It is interesting to note here that Einstein knew that the LT must depend explicitly on x' both because of the initial ansatz $\tau = \tau(x', y, z, t)$ and the explicit statement [34] concerning the necessary additional additive constants on the right sides of (E3.14)-(E3.17), noted in Section 2 above. However, by omitting the x -dependence of τ in the initial ansatz, the LT for the clock at B (with $x' = L$) was found to be the one that is actually valid for the clock at A (where $x' = 0$) and so does not correctly describe a synchronised clock at B. Such synchronised clocks actually exhibit the universal TD effect of Eqn(2.18), that is, there is no 'relativity of simultaneity' as implied if Eqns(6.6) and (6.7). Afterwards, in the applications of the LT in Ref. [2], the missing x' dependence of the final LT equations (E3.27)-(E3.30), for any synchronised clock that does not have $\xi = 0$, is forgotten¹¹, leading to the prediction, as will be seen below, of a spurious 'length contraction' effect in §4 of Ref. [2].

¹¹It was universally forgotten also throughout the 20th Century

7 §4. Physical Meaning of the Equations Obtained In Respect to Moving Rigid Bodies and Moving Clocks

Einstein first considers a rigid sphere of radius R at rest in S' with its centre at the origin. The equation of this sphere is

$$\xi^2 + \eta^2 + \zeta^2 = R^2 \quad (\text{E4.1})$$

Setting $t = 0$ in the LT (E3.27)-(E3.30) and using it to eliminate ξ , η and ζ in favour of x , y and z in Eqn(4.1) gives the equation:

$$\frac{x^2}{1 - (v/c)^2} + y^2 + z^2 = R^2 \quad (\text{E4.2})$$

which represents an ellipsoid of revolution. This however does not correctly describe the moving sphere as seen from S , since the LT (E3.27)-(E3.30) appropriate for $\xi = 0$ (the center of the sphere) has been also used to transform points on the surface of the sphere where it is no longer applicable.

Introducing spherical coordinates θ , ϕ the coordinates of an arbitrary point P on the sphere are:

$$\xi = R \sin \theta \cos \phi \quad (7.1)$$

$$\eta = R \sin \theta \sin \phi \quad (7.2)$$

$$\zeta = R \cos \theta \quad (7.3)$$

Assume that a clock, situated at the point P , is synchronised with a clock in S so that $\tau = t = 0$ when the center of the sphere is at the origin of coordinates in S . The LT giving the corresponding space-time coordinates in S is given, in Einstein's notation by Eqs(2.15), (2.16), (2.3) and(2.4) as:

$$\tau = \gamma(t - \beta(x - R \sin \theta \cos \phi)/c] \quad (7.4)$$

$$\xi - R \sin \theta \cos \phi = \gamma[x - R \sin \theta \cos \phi - vt] = 0 \quad (7.5)$$

$$\eta = R \sin \theta \sin \phi = y \quad (7.6)$$

$$\zeta = R \cos \theta = z \quad (7.7)$$

Combining (E4.1), (7.5), (7.6) and (7.7) the surface observed in S at time t is therefore:

$$(x - vt)^2 + y^2 + z^2 = R^2 \quad (7.8)$$

which is a sphere centered at $x = vt$. There is no therefore no shortening of the x -dimension of the sphere when the LT correctly describing a synchronised clock at any point on its surface, (7.4)-(7.7), is applied.

Einstein's error is to use the LT (E3.27)-(E3.30), that correctly describes a synchronised clock (i.e. one for which $\tau = 0$ when $t = 0$) at the origin of S' , that is the center of

the sphere, incorrectly to describe a synchronised clock at the point P on the surface of the sphere. At $t = 0$ Eqn(E3.27) gives

$$\tau_P = -\frac{\gamma\beta}{c}R \sin \theta \cos \phi \quad (7.9)$$

The time in S' varies with the position of P so that clearly (E3.27) does not describe a synchronised clock at P. The operation of clock synchronisation is a purely mechanical or electronic one, entirely under the control of the experimenter and independent of space-time physics. For example, clocks at different positions on the sphere can be synchronised by signals sent along equal-length cables connected to the centre of the sphere. When the latter coincides with the origin in S a triggering system sends synchronous signals along each cable to start the clocks, which each have an initial time offset to compensate for the signal delays [1]. Since the LT (E3.27)-(E3.30) does not describe such a synchronised clock when $\xi \neq 0$, the length contraction effect of Eqn(E4.2) is spurious. For further discussion of this point See Refs. [1, 4, 5, 7].

The most important, and correct, conclusion of Ref [2] concerning space-time physics is the time dilatation formula (E4.5) which is derived in an identical manner to Eqn(2.18) in Section 2 above. Note that, unlike for the discussion of length contraction, the derivation of the TD formula is not dependent on the value assigned to Einstein's parameter $x' = x(t = 0)$ (L in Eqns(2.15) and (2.16)). It is indeed precisely assigning incorrect values of x' to clocks on the surface of the sphere that results in the spurious prediction of length contraction.

After deriving the TD formula (6.1) Einstein discusses observations of times recorded by stationary or moving clocks. The case when a moving clock starts at the same position as the stationary one, moves along a curve (Einstein considers this curve as the limit of a polygonal line) and returns to its starting position, is now referred to as the 'Twin Paradox' [37] —less time has elapsed during the round trip according to observations of the moving clock as compared to the stationary one.

Einstein concludes §4 with the statement:

Thence we conclude that a balance-clock (Not a pendulum clock which is physically a system to which the Earth belongs. This case had to be excluded.) at the equator must go more slowly, by a small amount, than a precisely similar clock situated at one of the poles under otherwise identical conditions.

The correctness of the phrase 'go more slowly' depends on the frame from which the clock is being observed. Greater clarity would be obtained by replacing 'go more slowly' with 'appears to go more slowly when observed from the frame F than when it is observed in its own rest frame'. The question then arises: what are the frames F for which this statement is true? Einstein specifies an observer at a pole of the Earth, that is one that does not partake of the rotational motion of the Earth and a clock at the equator that does. Thus, to a very good approximation, the observer's frame F is an inertial one co-moving with the centroid of the Earth, whereas the equator clock is in an accelerated frame.

Only special relativistic effects were considered by Einstein in Ref. [2]. In fact, due to the rotation of the Earth, its surface bulges outward at the equator, which places an observer there at a higher gravitational potential¹² than one at the pole. It turns out [38, 39, 40] that, for example, for an atomic clock, the increase of frequency of equator clock photons observed at the pole due to gravitational blue shift almost exactly compensates TD (the transverse Doppler effect) for an observer at a pole. Indeed, as shown in Refs. [39, 40], this cancellation between TD and gravitational frequency shift occurs for any pair of clocks situated on the Earth's geoid—a surface at sea-level perpendicular to the direction of a local plumb line which is a equipotential surface of the combined gravitational and centrifugal potential.

Einstein's exclusion from consideration of pendulum clocks in the English translation of Ref. [2] means that he must have later considered the importance of gravitational effects, if not already gravitational frequency shifts, at least the dependence of the period of a pendulum on the size of the ambient gravitational field.

8 §5. The Composition of Velocities

Einstein derived the composition laws for longitudinal and transverse velocity components by simple substitution of the relations:

$$\xi = \omega_\xi \tau, \quad \eta = \omega_\eta \tau, \quad \zeta = 0 \quad (\text{E5.1})$$

in the LT equations (E3.27)-(E3.30). This leads immediately to the relations:

$$x = \frac{(\omega_\xi + v)t}{1 + \frac{v\omega_\xi}{c^2}} \quad (\text{E5.2})$$

$$y = \omega_\eta t \frac{\sqrt{1 - (v/c)^2}}{1 + \frac{v\omega_\xi}{c^2}} \quad (\text{E5.3})$$

Specialising to the case of motion in S' parallel to the ξ -axis (i.e. $\omega_\eta = 0$) and setting $x = Vt$ (E5.2) yields the parallel velocity addition relation (PVAR)

$$V = \frac{v + \omega}{1 + \frac{v\omega}{c^2}} \quad (\text{E5.9})$$

where

$$\omega^2 = \omega_\xi^2 + \omega_\eta^2 \quad (\text{E5.6})$$

Einstein derived the PVAR directly from the LT, but this relation may also be obtained assuming only Einstein's second postulate [41, 42] or, alternatively, by assuming that V is a single-valued function of v and ω and using the Reciprocity Principle [32].

The PVAR really involves three inertial frames S, S' and S₀ where the last is the proper frame of the moving object. Even so, Einstein's derivation uses only the LT

¹²The gravitational potential is defined to be negative and to vanish at infinity

(E3.27)-(E3.30) connecting the frames S and S'. To understand the operational meaning of the symbols used in this derivation it is convenient to revert to the notation of Section 2. The observer in the thought experiment corresponding to the derivation of the PVAR is at rest in the frame S and the PVAR gives the velocity of the object relative to this observer, V , in the case that the velocity of the object relative to an observer at rest in the frame S' is ω . The appropriate LT is therefore (2.15) and (2.16) with $L = 0$ and $x' = \omega\tau'(C') = \omega t'(S)$, $x = V\tau$:

$$\omega t'(S) = \gamma[V - v]\tau \quad (8.1)$$

$$t'(S) = \gamma[1 - \frac{vV}{c^2}]\tau \quad (8.2)$$

where the equivalence of Eqn(2.31) between the apparent time interval $t'(S)$ and a proper time interval $\tau'(C')$ has been used. In these equations, whereas τ is a proper time recorded by a clock at rest in S, $t'(S)$ is the apparent time of a clock at rest in S' as viewed from S. The ratio of (8.1) to (8.2) is:

$$\omega = \frac{V - v}{1 - \frac{vV}{c^2}} \quad (8.3)$$

which gives, after rearrangement, (E5.9).

If $t_0(S)$ is the apparent time of a clock at rest in S_0 (i.e. in the proper frame of the moving object), as viewed from S, the TD relation for an observer in the frame S is:

$$\tau = \gamma_V t_0(S) \quad (8.4)$$

while that for an observer in the frame S' is:

$$\tau' = \gamma_\omega t_0(S') \quad (8.5)$$

where

$$\gamma_v \equiv \frac{1}{\sqrt{1 - \beta_v^2}} \equiv \gamma, \quad \beta_v \equiv \frac{v}{c} \equiv \beta$$

Considering now observations of events specified in the frame S' or S_0 , from S, the replacements $\tau' \rightarrow t'(S)$ and $t_0(S') \rightarrow t_0(S)$ should be made in (8.5) giving

$$t'(S) = \gamma_\omega t_0(S) \quad (8.6)$$

Substitution of (8.4) and (8.6) into (8.1) and (8.2) gives:

$$\gamma_\omega \beta_\omega = \gamma[\gamma_V \beta_V - \beta \gamma_V] \quad (8.7)$$

$$\gamma_\omega = \gamma[\gamma_V - \beta \gamma_V \beta_V] \quad (8.8)$$

These equations are the LT between the frames S and S' of the spatial and temporal components, respectively, of the dimensionless velocity 4-vector $(\gamma_V; \gamma_V \beta_V, 0, 0)$. These formulae will be found useful in discussing the transformation properties of electric and magnetic fields in the following section.

A more transparent method to derive the PVAR, mentioned by Einstein at the end of §5, is by successive application of the LT (E3.27)-(E3.30), that is by invoking the group property of the LT. However, no such calculation was presented.

9 §6. Transformation of the Maxwell-Hertz Equations for Empty Space. On the Nature of the Electromotive Forces in a Magnetic Field During Motion

In the first part of this section Einstein derives the transformation laws for electric and magnetic fields in ‘empty space’ from the assumed covariance of Maxwell’s equations obtaining the well-known results, in modern notation and Gaussian units:

$$E_\xi = E_x, \quad B_\xi = B_x \quad (\text{E6.13})$$

$$E_\eta = \gamma[E_y - \beta B_z], \quad B_\eta = \gamma[B_y + \beta E_z] \quad (\text{E6.14})$$

$$E_\zeta = \gamma[E_z + \beta B_y], \quad B_\zeta = \gamma[B_z - \beta E_y] \quad (\text{E6.15})$$

The fields in these equations are ‘force fields’, that, when substituted into the Lorentz force equation

$$\frac{d\vec{p}}{dt} = \vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B}) \quad (9.1)$$

where $\vec{p} \equiv \gamma\vec{\beta}mc$ is the relativistic 3-momentum, give the force \vec{F} on a test charge q at the field point under consideration. The fields of §8 of the paper although represented by identical symbols are actually ‘radiation fields’ that provide the classical description of the creation, propagation and destruction of real photons. However in §9 and §10 force fields are again considered.

As will be seen shortly, a certain tacit assumption concerning the nature of electric and magnetic fields underlies the transformation equations (E6.13)-(E6.15). To understand this assumption it will be found convenient to write the transformation equations in tensor notation. To render the equations more transparent the notation of Section 2 for space-time coordinates in the frame S’ (x', y', z', t') instead of Einstein’s (ξ, η, ζ, τ) is used. The electromagnetic field tensor $F^{\alpha\beta}$ is defined by the equations:

$$-E^i \equiv F^{0i} \equiv \partial^0 A^i - \partial^i A^0 \quad (9.2)$$

$$-B^i \equiv \epsilon_{ijk} F^{jk} \equiv \epsilon_{ijk} (\partial^j A^k - \partial^k A^j) \quad (9.3)$$

where A^α is the 4-vector electromagnetic potential. The alternating tensor, ϵ_{ijk} , is equal to $+1(-1)$ for even(odd) permutations of ijk . Greek indices α, β, \dots take the values 0, 1, 2, 3 and latin indices i, j, \dots take the values 1, 2, 3. Repeated upper and lower indices in (9.3) are summed over 1, 2 and 3. The contravariant space-time 4-vector x^α is defined according to the relations:

$$(x^0 = x_0, x^1, x^2, x^3) \equiv (ct, x, y, z) \quad (9.4)$$

The following notation for partial derivatives is also used:

$$\partial^0 \equiv \frac{\partial}{\partial x^0} = \frac{1}{c} \frac{\partial}{\partial t} \quad (9.5)$$

$$\partial^i \equiv -\frac{\partial}{\partial x^i} \equiv -\nabla_i \quad (9.6)$$

With this notation the transformation laws (E6.13)-(E6.15) are all subsumed in the single tensor equation:

$$F'^{\alpha\beta} = \frac{\partial x'^{\alpha}}{\partial x^{\gamma}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} F^{\gamma\delta} \quad (9.7)$$

where repeated upper and lower indices are summed over 0,1,2 and 3. In (9.7) the values of the partial derivatives are derived from the infinitesimal LT between the frames S and S':

$$dx' = \gamma(dx - \beta dx^0) \quad (9.8)$$

$$dx'^0 = \gamma(dx^0 - \beta dx) \quad (9.9)$$

$$dy' = dy, \quad dz' = dz \quad (9.10)$$

The values of the partial derivatives in (9.7) that give the transformation laws (E6.13)-(E6.15) require a certain assumption, concerning the space-time functional dependence of the fields, that will now be discussed. This assumption is that all components of the electric and magnetic fields are 'classical' ones in the sense that all space-time coordinates may be considered to be independent variables. If this is the case, then the chain rule of differential calculus is valid for any generic field component F [43]:

$$\begin{aligned} dF &= \frac{\partial F}{\partial x^0} dx^0 + \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy + \frac{\partial F}{\partial z} dz \\ &= \frac{\partial F}{\partial x'^0} dx'^0 + \frac{\partial F}{\partial x'} dx' + \frac{\partial F}{\partial y'} dy' + \frac{\partial F}{\partial z'} dz' \end{aligned} \quad (9.11)$$

The transformation law between ∂^{α} and ∂'^{α} is found by using the inverse equations of (9.8)-(9.10) to eliminate $d\vec{x}$ and dx^0 in favour of $d\vec{x}'$ and dx'^0 in Eqn(9.11). Collecting terms and equating the coefficients of the three components of $d\vec{x}'$ and of dx'^0 in the second and third members of (9.11) gives, since the field F is arbitrary, the transformation laws of the partial differential operators as:

$$\frac{\partial}{\partial x'} = \gamma \left(\frac{\partial}{\partial x} + \beta \frac{\partial}{\partial x^0} \right) \quad (9.12)$$

$$\frac{\partial}{\partial x'^0} = \gamma \left(\frac{\partial}{\partial x^0} + \beta \frac{\partial}{\partial x} \right) \quad (9.13)$$

$$\frac{\partial}{\partial y'} = \frac{\partial}{\partial y}, \quad \frac{\partial}{\partial z'} = \frac{\partial}{\partial z} \quad (9.14)$$

These equations show that the partial derivatives transform as a 4-vector under the LT. In virtue of the definitions (9.2) and (9.3) of $F^{\alpha\beta}$ and the 4-vector character of A^{α} , the electric and magnetic fields then transform as a second rank tensor according to Eqn(9.7). The transformation coefficients in (9.7) then have the values, obtained from (9.8)-(9.10):

$$\frac{\partial x'}{\partial x} = \gamma, \quad \frac{\partial x'}{\partial y} = \frac{\partial x'}{\partial z} = 0, \quad \frac{\partial x'}{\partial x^0} = -\gamma\beta \quad (9.15)$$

$$\frac{\partial x'_0}{\partial x} = -\gamma\beta, \quad \frac{\partial x'_0}{\partial y} = \frac{\partial x'_0}{\partial z} = 0, \quad \frac{\partial x'_0}{\partial x^0} = \gamma \quad (9.16)$$

$$\frac{\partial y'}{\partial x} = 0, \quad \frac{\partial y'}{\partial y} = 1, \quad \frac{\partial y'}{\partial z} = 0, \quad \frac{\partial y'}{\partial x^0} = 0 \quad (9.17)$$

$$\frac{\partial z'}{\partial x} = 0, \quad \frac{\partial z'}{\partial y} = 0, \quad \frac{\partial z'}{\partial z} = 1, \quad \frac{\partial z'}{\partial x^0} = 0 \quad (9.18)$$

These coefficients, when substituted into (9.7) give exactly Einstein's transformation laws (E6.13)-(E6.15). It is important to stress again that, in order to derive (9.15)-(9.18) the field components F are required to respect the chain rule relation (9.11). This requires, in particular, that the x -coordinate and the time must be independent variables.

In reality, however the force fields in 'empty space' considered by Einstein do not exist. Since they are defined by the force they exert on a test charge they can be non-zero only if, in the problem, there is at least one source charge to produce the fields. The space in the region of the test charge therefore cannot be 'empty'. The simplest possible electrodynamic system described by force fields therefore contains two electric charges, one of which may be considered as the 'source' of the fields that act on the other 'test' charge, although it is clear, from the symmetry of the system, that these roles may be interchanged. Here, the simplest case of a single source charge in uniform motion will be considered. The corresponding electric and magnetic fields of pre-relativistic classical electromagnetism (CEM) are given by the equations [20]:

$$\vec{E}(\text{CEM}) = \frac{Q\vec{r}}{r^3\gamma_u^2(1 - \beta_u^2 \sin^2 \psi)^{\frac{3}{2}}} \quad (9.19)$$

$$\vec{B}(\text{CEM}) = \frac{\vec{u} \times \vec{E}}{c} \quad (9.20)$$

whereas, in RCED, they are given by [8]:

$$\vec{E}(\text{RCED}) = \frac{Q}{r^2} \left[\frac{\hat{i} \cos \psi}{\gamma_u} + \hat{j} \gamma_u \sin \psi \right] \quad (9.21)$$

$$\vec{B}(\text{RCED}) = \frac{\vec{u} \times \vec{E}}{c} \quad (9.22)$$

In these equations the velocity of the source charge Q is $\vec{u} = \hat{i}\beta_u c$ and the radius vector is $\vec{r} = r(\hat{i} \cos \psi + \hat{j} \sin \psi)$ where \hat{i} , \hat{j} and \hat{k} are unit vectors directed along the x , y , z axes. In (9.19)-(9.22) \vec{r} is specified at the instant at which the fields are defined, although the Heaviside formula (9.19) is commonly derived [44] from retarded Liénard-Wiechert potentials [21]. As mentioned in Section 2 above, the pre-relativistic Heaviside formulae (9.19), (9.20) are actually incorrect, but what is important for the present discussion is only the functional space-time dependence of the fields in (9.19)-(9.22) and of the corresponding 4-vector potential A^α in the defining equations of the fields (9.2) and (9.3). In all of these equations, the field point, \vec{x} , always appears in combination with the vector, \vec{x}_Q , specifying the instantaneous position of the source charge, to define the radius vector, \vec{r} , giving the relative positions of the source and test charges:

$$\vec{r} \equiv \vec{x} - \vec{x}_Q \quad (9.23)$$

When the source charge is in motion, the functional time dependence of the fields is implicit in that of the source charge position $\vec{x}_Q = \vec{x}_Q(t)$. Because A^α and all the fields are functions of $r = |\vec{x} - \vec{x}_Q(t)|$ it follows, for the case of Eqns(9.19)-(9.22) where the source charge is in motion parallel to the x -axis, that x and t are no longer independent variables, as assumed in Eqn(9.11), but instead their partial derivatives satisfy the relation [11]:

$$\left. \frac{\partial}{\partial t} \right|_x = -u \left. \frac{\partial}{\partial x} \right|_t \quad (9.24)$$

Also, as demonstrated in Ref. [11] r is a Lorentz-invariant quantity so that in virtue of Eqns(9.14), and in contrast to Eqn(9.12), the transformation of the x partial derivative is, like the y and z partial derivatives, time independent:

$$\frac{\partial}{\partial x'} = \frac{\partial}{\partial x} \quad (9.25)$$

A consequence of (9.24) and (9.25) is that the x and t partial derivatives do not transform as components of a 4-vector according to Eqns(9.12) and (9.13) so that the electric and magnetic fields produced by a uniformly moving charge do not transform, according to (9.7), as components of a second-rank tensor.

The actual transformation laws of the RCED fields of (9.21) and (9.22) are easily read off from these equations. Denoting by \vec{E}^* the electric field in the rest frame S^* of the source charge Q , (9.21) gives:

$$E_x = \frac{E_x^*}{\gamma_u}, \quad E_y = \gamma_u E_y^* \quad (9.26)$$

$$B_z = \beta_u E_y = \beta_u \gamma_u E_y^* \quad (9.27)$$

For comparison with Einstein's transformation equations (E6.13)-(E6.15), suppose that the velocity of the source charge in the frame S' is w . Then (9.21) and (9.22) give, for the fields in the frame S' :

$$\vec{E}' = \frac{Q}{r^2} \left[\hat{i} \cos \psi + \hat{j} \gamma_w \sin \psi \right] \quad (9.28)$$

$$\vec{B}' = \frac{\vec{w} \times \vec{E}'}{c} \quad (9.29)$$

Considering the transformations similar to (9.26) and (9.27) between S^* and S' , and eliminating the S^* fields between these equations and (9.26) and (9.27) gives, for the field transformations from S to S' :

$$E'_x = \frac{\gamma_u}{\gamma_w} E_x, \quad E'_y = \frac{\gamma_w}{\gamma_u} E_y, \quad B'_z = \frac{\beta_w \gamma_w}{\beta_u \gamma_u} B_z \quad (9.30)$$

Since, from Eqn(9.27), $E_y = B_z/\beta_u$ the second of Eqns(9.30) may be written as

$$E'_y = \frac{\gamma_w}{\beta_u \gamma_u} B_z \quad (9.31)$$

Because the velocities w , u and v satisfy the PVAR:

$$\beta_w = \frac{\beta_u - \beta}{1 - \beta_u \beta} \quad (9.32)$$

the temporal component of the dimensionless 4-vector velocity $U = (\gamma_u; \gamma_u \beta_u, 0, 0)$ transforms between S and S' (see Eqn(8.8) as:

$$\gamma_w = \gamma(\gamma_u - \beta \gamma_u \beta_u) \quad (9.33)$$

Combining (9.31) and (9.33) then gives

$$E'_y = \frac{\gamma}{\beta_u} (B_z - \beta \beta_u B_z) = \gamma \left(\frac{B_z}{\beta_u} - \beta B_z \right) = \gamma (E_y - \beta B_z) \quad (9.34)$$

The Lorentz transformation of the spatial component of U between S and S' is (see Eqn(8.7)):

$$\gamma_w \beta_w = \gamma(\gamma_u \beta_u - \beta \gamma_u) \quad (9.35)$$

Combining (9.35) with the last equation in (9.30) gives:

$$B'_z = \frac{\gamma}{\beta_u}(\beta_u B_z - \beta B_z) = \gamma(B_z - \frac{\beta B_z}{\beta_u}) = \gamma(B_z - \beta E_y) \quad (9.36)$$

Eqns(9.34) and (9.36) for the transformation of the transverse electric field and the magnetic field agree with the corresponding equations in (E6.14) and (E6.15). Although the tensor transformation law is not valid, it gives, fortuitously, the correct transformation laws for these fields. Since the transformation coefficients of (9.34) and (9.36) depend only on the relative velocity of the frames S and S' , and are independent of the velocity of the source charge, these equations are covariant. It is otherwise with the transformation law of the longitudinal electric field in the first of Eqns(9.30) which does not agree with Einstein's covariant transformation law in (E6.13), $E'_x = E_x$. In this case the transformation coefficient does depend on the velocity of the source charge. There is therefore a preferred frame (the frame in which the source charge is at rest) in the problem. Because of this, relativistic covariance is broken. Fundamentally, the tacit assumption underlying Einstein's analysis, introduced into theoretical electromagnetism by Faraday and Maxwell, that electric and magnetic forces may be correctly described by *local classical fields* is not true of the actual force fields of a uniformly moving charge. This applies both to the pre-relativity fields of Eqns(9.19) and (9.20) and the RCED fields of (9.21) and (9.22). The actual force fields are produced by the source charge and the motion of this charge affects, instantaneously, the force experienced by the test charge. Note that that this remains true for the 'present time' Heaviside formulae (9.19) and (9.20) even though the corresponding interaction is retarded. All inertial frames are not equivalent, so that special relativistic invariance breaks down in the description of inter-charge forces.

Another consequence of this breakdown of covariance is the frame dependence of some of the Maxwell equations [11]. For example, the Ampère law equation in the first of Eqns(E6.1):

$$\frac{1}{c} \frac{\partial E_x}{\partial t} = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \quad (9.37)$$

is replaced by the non-covariant equation

$$\frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{Q\beta_u}{r^3} \left(\gamma_u - \frac{1}{\gamma_u} \right) (2 - 3 \sin^2 \psi) = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \quad (9.38)$$

This is demonstrated by direct calculation of the partial derivatives of the fields of (9.21) and (9.22), taking into account the relation (9.24) connecting the x and t partial derivatives. A similar calculation of the y - and z -components of Ampère's law show that the first equations of (E6.11) and (E6.12) remain valid. In contrast to the Ampère law, all components of the Faraday-Lenz law in the second equations in (E6.10)-(E6.12) remain valid for the RCED force fields of Eqns(9.21) and (9.22) and the CEM fields of (9.19) and (9.20). This follows directly from the defining equations of the fields in terms of A^α , (9.2) and (9.3), written in 3-vector notation as:

$$\vec{B} = \vec{\nabla} \times \vec{A}, \quad \vec{E} = -\vec{\nabla} A^0 - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \quad (9.39)$$

Taking the time derivative of the first of these equations and substituting the value of $\partial \vec{A}/\partial t$ from the second equation gives:

$$\frac{1}{c} \frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times (\vec{\nabla} A^0) - \vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \vec{E} \quad (9.40)$$

in virtue of the 3-vector identity $\vec{\nabla} \times (\vec{\nabla} \phi) = 0$, for arbitrary ϕ . Thus the Faraday-Lenz law is valid in any inertial frame (is covariant) independently of the space-time functional dependence of A^α or the force fields.

Another consequence of the non-covariant character of the electric force fields is the breakdown of the electric Gauss law if the source charge is in motion [9, 11]:

$$\vec{\nabla} \cdot \vec{E} = \frac{Q}{r^3} \left(\gamma_u - \frac{1}{\gamma_u} \right) (2 - 3 \sin^2 \psi) \quad (9.41)$$

The appearance of a similar covariance-breaking term in the Gauss law (9.41) and the Ampère law (3.38) follows from Eqn(9.24) and the first member of Eqn(9.27). Since the electric field in Eqn(9.19) or (9.21) is confined to the x - y plane, and only the z -component of the magnetic field is non-vanishing:

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = \frac{1}{\beta_u} \left[\beta_u \frac{\partial E_x}{\partial x} + \frac{\partial(\beta_u E_y)}{\partial y} \right] \\ &= \frac{1}{\beta_u} \left[-\frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial B_z}{\partial y} \right] \end{aligned} \quad (9.42)$$

or

$$-\frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial B_z}{\partial y} = \beta_u \vec{\nabla} \cdot \vec{E} \quad (9.43)$$

So that if Gauss law is valid $\vec{\nabla} \cdot \vec{E} = 0$ then so is the x -component of the Ampère law (the first of Eqns(E6.1) with $B_y = 0$). In the case of the field in (9.41) where the Gauss law is not respected (9.38) follows directly from (9.41) and (9.43) on setting B_y to zero.

As shown on Ref [11] the wave equations of all components of the electric and magnetic fields derived from Eqns(9.21) and (9.22) are also modified by the addition of β_u dependent covariance-breaking terms, so that Maxwell's derivation of 'electromagnetic waves' associated with these fields also breaks down. The leading covariance breaking terms in Gauss' law and the electric field wave equations are of $O(\beta_u^4)$, and for the Ampère law and the magnetic field wave equations of $O(\beta_u^5)$, so that no practical applications of Maxwell's equations in the non-relativistic domain are affected significantly by these modifications. However it is clear that the heretofore fundamental physical status of Maxwell's equations is profoundly modified by the the lack of covariance of the Ampère law and the Gauss' law for electric fields.

In the second part of §6 Einstein returns to the general problem of the relationship between electric and magnetic forces that was discussed for the special case of electromagnetic induction in the Introduction. Einstein's remarks here are important, both for what is said, and for what is left unsaid:

1. If a unit electric point charge is in motion in an electromagnetic field there acts upon it, in addition to the electric force, an

‘‘electromotive force’’ which, if we neglect the terms multiplied by the second and higher powers of v/c , is equal to the vector-product of the velocity of the charge and the magnetic force divided by the velocity of light. (Old manner of expression).

2. If a unit electric point charge is in motion in an electromagnetic field, the force acting upon it is equal to the electric force which is present at the locality of the charge, and which we ascertain by transformation of the field to a system of co-ordinates at rest relatively to the electrical charge (New manner of expression).

The analogy holds for ‘‘electromotive forces’’. We see that electromotive force plays in the developed theory merely the part of an auxiliary concept which owes its introduction to the circumstance that electric and magnet forces do not exist independently of the state of motion of the system of coordinates.

Furthermore it is clear that the asymmetry mentioned in the introduction as arising when we consider the currents produced by the relative motion of a magnet and a conductor now disappears. Moreover, questions as to the ‘‘seat’’ of electrodynamic electromotive forces (unipolar machines) now have no point.

Einstein clearly states here that the essential physical phenomenon is the force on the test charge and that the electric and magnetic ‘fields’ serve only as labels for different terms in the force equation, the relative contributions of which vary according to the inertial frame in which the force is evaluated. It is pointed out that the ‘‘electromotive force’’ (Lorentz force due to motion in a magnetic field) that acts on the test charge may be annulled by transformation into the rest frame of the test charge, in which only the force associated with the electric field acts. Einstein later uses this argument in §10 to derive, correctly to lowest order in β , the Lorentz force from the transformation laws of electric and magnetic fields. Because it can be ‘transformed away’ in this manner the Lorentz force (and by implication the ‘magnetic field’ itself) is assigned the status of an ‘auxiliary concept’. But by the same token the electric field is also an ‘auxiliary concept’ in that it, like the magnetic field, is completely specified by the configuration of electric charges that constitute its source. Throughout Ref [2] Einstein discusses only fields and either their interactions with test charges or the description they provide of electromagnetic radiation. In what manner these fields are produced by their sources, which is essential for a complete physical description, is not even mentioned. The covariance of Maxwell’s equations with sources is discussed in §9 but not the physical interpretation of the fields and sources, which, in their most important physical application, the description of radiation, is quite different to that of the force fields discussed in §6, §9 and §10.

In fact since all mechanical effects on a test charge are described by Eqn(3.4) (or by its generalisation to the case of multiple source charges) in which only the physical parameters specifying the spatial and kinematical configuration of the source charges appear, both the electric and magnetic ‘fields’, that serve only as labels for the different terms on the right side of (3.4), are inessential ‘auxiliary concepts’, or alternatively, ‘mathematical abstractions’. The same is true of the 4-vector potential, A^α , in terms of

which, electric and magnetic fields are defined by Eqns(9.2) and (9.3). This, in turn, is unambiguously specified ¹³ by the physical parameters that describe the configuration of source charges [8]. This is shown in Fig.1. All force predictions are contained in the ‘Intercharge Force’; box inside the dot-dashed line. They are obtained without the necessity to introduce any ‘field’ concept whatever.

At the end of §6, Einstein returns to the problem of induction, as analysed in different inertial frames discussed in the Introduction and considers it solved by the comments in the paragraphs 1. and 2. cited above. If he had actually calculated the electric field in the rest frame of the test charge for the configuration shown in Fig.1b, using the then-available Heaviside equation with a radial electric field he would have found, as explained in Section 2 above, that the induction force is not the same as the Lorentz force in Fig5a, but actually vanishes! As described in [10] consistent results in the two frames are given by the RCED formulae (9.21) and (9.22). The calculation shows that the induction force is, however, not exactly equal in the frames S and S', as stated by Einstein, but differs by terms of $O(\beta_u^4)$. The origin of this difference is the source velocity dependence of the electric field in Eqn(9.21), which breaks covariance for the longitudinal component of the electric field.

The closing remark of §6 concerns the absence, in electromagnetism, of magnetic monopoles. Since the Gauss law for the magnetic field $\vec{\nabla} \cdot \vec{B} = 0$ follows from the definition of the magnetic field in terms of \vec{A} in first of equations (9.39) and the three-vector identity $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$ this comment of Einstein's is certainly well-founded.

10 §7. Theory of Doppler's Principle and Aberration

Einstein first writes down in Eqns(E7.1)-(E7.3) the field components, in the frame S, of an electromagnetic wave propagating in the direction defined by the direction cosines l, m, n relative to the x, y, z axes. The phase of the wave in S is written as:

$$\Phi = \omega \left\{ t - \frac{1}{c}(lx + my + nz) \right\} \quad (\text{E7.4})$$

Similarly the phase of the wave in S' is written as:

$$\Phi' = \omega' \left\{ \tau - \frac{1}{c}(l'\xi + m'\eta + n'\zeta) \right\} \quad (\text{E7.8})$$

At this point there is a logical hiatus in the discussion. Einstein immediately writes down the transformation equations giving the quantities ω', l', m' and n' in terms of ω, l, m and n :

$$\omega' = \omega \gamma (1 - \beta l) \quad (\text{E7.9})$$

$$l' = \frac{l - \beta}{1 - \beta l} \quad (\text{E7.10})$$

¹³A particular gauge, Lorenz gauge [45], is a necessary consequence of the postulates from which Eqn(3.4) is derived. Thus there is no freedom of the choice of gauge used to define A^α in RCED.

$$m' = \frac{m}{\gamma(1 - \beta l)} \quad (\text{E7.11})$$

$$n' = \frac{n}{\gamma(1 - \beta l)} \quad (\text{E7.12})$$

The field components are also transformed into the frame S' in Eqn(E7.5)-(E7.7), but the actual values of the field components are irrelevant for the discussion of the Doppler effect and aberration in this section.

In order to derive (E7.9)-(E7.12) another important hypothesis, not mentioned by Einstein, is needed: $\Phi' = \Phi$. That is, it must be assumed that the phase of the wave is a Lorentz invariant quantity. Presumably Einstein did this, used the LT (E3.27)-(E3.30) to replace τ, ξ, η and ζ in (E7.8) by t, x, y and z , and equating coefficients of these variables in the thus-transformed Eqn(E7.8) with those in Eqn(E7.4) arrived (after some algebraic manipulation) at Eqns(E7.9)-(E7.12). This method does yield the correct Doppler effect and aberration formulae, but, as discussed below, poses a certain problem of mathematical logic.

From a modern viewpoint The postulate of the invariance of Φ and the fact that $X \equiv (ct = x^0; x, y, z)$ is a 4-vector, implies that $\Omega \equiv (\omega/c; \omega l/c, \omega m/c, \omega n/c)$ must also be a 4-vector so that (E7.4) may be written as:

$$\Phi = \Omega \cdot X \quad (\text{10.1})$$

Since Ω is a unidimensional 4-vector its LT between the frames S and S' is given by the space-time symmetric LT [33]:

$$\Omega'_0 = \gamma(\Omega_0 - \beta\Omega_x) \quad (\text{10.2})$$

$$\Omega'_x = \gamma(\Omega_x - \beta\Omega_0) \quad (\text{10.3})$$

$$\Omega'_y = \Omega_y, \quad \Omega'_z = \Omega_z \quad (\text{10.4})$$

or

$$\omega' = \omega\gamma(1 - \beta l) \quad (\text{10.5})$$

$$\omega' l' = \omega\gamma(l - \beta) \quad (\text{10.6})$$

$$\omega' m' = \omega m \quad (\text{10.7})$$

$$\omega' n' = \omega n \quad (\text{10.8})$$

Where factors $1/c$ on both sides of (10.5)-(10.8) have been divided out. Eqn(10.5) is identical to (E7.9) and yields (E7.11) and (E7.12) on eliminating ω/ω' from (10.7) and (10.8). Eqn(E7.10) follows on eliminating ω/ω' between (10.5) and (10.6).

The problem of mathematical logic in Einstein's surmised proof of Eqns(E7.9)-(E7.12) arises from the factor in curly brackets in (E7.4). The quantity $lx + my + nz$ is a rotational invariant that has the value:

$$s = \sqrt{x^2 + y^2 + z^2} \quad (\text{10.9})$$

The distance s is the displacement of the wave front (i.e. a surface of constant phase) in its direction of propagation during the time t . Thus, (E7.4) may be written as:

$$\Phi = \omega(t - \frac{s}{c}) \quad (\text{10.10})$$

Since the wave front moves at speed c , $s = ct$ and (10.10) gives $\Phi = 0$. Since the field components in Eqns(E7.1)-(E7.3) are all proportional to $\sin \Phi$ they all vanish identically. So Einstein's electromagnetic wave (as defined) does not exist! In a similar way, (E7.8) may be written as:

$$\Phi' = \omega(\tau - \frac{s'}{c}) \quad (10.11)$$

Since, by the second postulate, the wave front also moves at speed c in S' , $s' = c\tau$ and $\Phi' = 0$. Thus (10.10) and (10.11) and the Lorentz invariance of the phase $\Phi = \Phi'$ reduce to the equation:

$$\Phi = \omega \times 0 = \omega' \times 0 = \Phi' \quad (10.12)$$

which is true for any finite values of ω and ω' . This would seem to indicate that Einstein's presumed derivation of (E7.9)-(E7.12) by (tacitly) setting Φ equal to Φ' , and Lorentz transforming one set of space-time coordinates, was a mathematically hazardous enterprise that finally 'fell on its feet'!

At the end of §7 Einstein writes down, without derivation, two equations for the transformation of the 'intensity' of an electromagnetic wave:

$$(A')^2 = \frac{A^2(1 - \beta \cos \phi)^2}{1 - \beta^2} \quad (E7.17)$$

$$(A')^2 = \frac{A^2(1 - \beta)}{1 + \beta} \quad (E7.18)$$

The quantities A and A' are called 'the amplitudes of the electric or magnetic force' which shows some confusion over the physical meaning of the field components in (E7.1)-(E7.3) and (E7.5)-(E7.7). These are radiation fields, not force fields. The squares of these radiation fields give the energy density ρ_E in the electromagnetic wave according to the well-known formula:

$$\rho_E = \frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2) \quad (10.13)$$

As pointed out above, as defined, Eqns(E7.1)-(E7.3) and Eqns(E7.5)-(E7.7) imply $A = A' = 0$. This problem is removed by making the permissible substitutions $\sin \Phi \rightarrow \cos \Phi$ and $\sin \Phi' \rightarrow \cos \Phi'$ in these equations. However, (E7.17) and (E7.18) derived from the transformation of force fields in Eqns(E7.6) and (E7.7) still make no sense physically. This can be seen by noting that a 'plane electromagnetic wave' is in fact a beam of monochromatic photons [14]. The transformation of the energy density of the wave may therefore be derived from the relativistic kinematics of photons. Since the energy density $\rho_E \simeq A^2$ is the product of the photon energy E_γ and the photon number density n_γ , which is Lorentz invariant, the transformation law of ρ_E may be derived from that of the photon energy. This gives, instead of (E7.17) and (E7.18):

$$\rho'_E = \rho_E \frac{1 - \beta \cos \phi}{\sqrt{1 - \beta^2}} \quad (10.14)$$

$$\rho'_E = \rho_E \sqrt{\frac{1 - \beta}{1 + \beta}} \quad (10.15)$$

Notice that the transformation law of the photon energy, and hence of the energy density of the beam, is the same as that of the photon frequency in Eqns(E7.13) and (E7.14) not that of the square of the force fields in Eqns(E7.6) and (E7.7). This is a consequence

of the Planck-Einstein relation, $E_\gamma = h\nu_\gamma$, to be further discussed in the following section. Einstein's error in deriving Eqns(E7.17) and (E7.18) is the misidentification of force fields, describing the effects of space-like virtual photon exchange between electric charges, with the radiation fields appropriate to the description of the monochromatic beams of real photons, which manifest the Doppler effect and aberration. The distinction between radiation fields and force fields is further discussed in Section 12 below.

Einstein's final statement in §7 :

It follows from these results that to an observer approaching the source of light with velocity c , this source of light must appear of infinite intensity.

remains true according to (10.15). although the approach to 'infinite intensity' is less rapid than in Eqn(E7.18).

11 §8. Transformation of the Energy of Light Rays. Theory of the Pressure of Radiation Exerted on Perfect Reflectors

In this section Einstein discusses, firstly, the transformation law of the energy density of an electromagnetic wave and, secondly, the radiation pressure due to a plane electromagnetic wave at oblique incidence on a uniformly moving plane mirror. Since plane electromagnetic waves may be identified with a parallel beam of monochromatic photons, all the results of this section may, alternatively, be obtained in a simple and transparent manner by invoking the relativistic kinematics of real photons.

To obtain the energy transformation law of a 'light ray' Einstein considers a spherical surface in the frame S with the equation:

$$(x - lct)^2 + (y - mct)^2 + (z - nct)^2 = R^2 \quad (\text{E8.1})$$

This equation represents a sphere of radius R centered at $x = lct$, $y = mct$, $z = nct$. The center moves in the same direction and at the same speed as the plane wave considered in §7. Even though the plane wave is of infinite extent in all three dimensions, Einstein considers only the energy of the 'light complex' (Lichtcomplexe) inside the spherical surface. The spurious 'length contraction' in the direction of motion of the spherical surface in S as viewed from S' according to (E4.2) is now invoked to claim that the surface (E8.1), as observed in S' at $\tau = 0$ is:

$$\xi^2 \frac{(1 - \beta l)^2}{1 - \beta^2} + (\eta - m\gamma\beta\xi)^2 + (\zeta - n\gamma\beta\xi)^2 = R^2 \quad (\text{E8.2})$$

The ratio of the volume, V , inside the sphere (E8.1) to that, V' , inside the ellipsoid of revolution (E8.2) is:

$$\frac{V'}{V} = \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \phi} \quad (\text{E8.3})$$

Setting the total wave energy inside (E8.1) equal to $A^2V/8\pi$, that inside (E8.2) to $(A')^2V'/8\pi$ and using (E7.17) for the ratio $(A')^2/A^2$ the transformation law of the energy of the ‘light complex’ is found to be:

$$\frac{E'}{E} = \frac{(A')^2V'}{A^2V} = \frac{1 - \beta \cos \phi}{\sqrt{1 - \beta^2}} \quad (\text{E8.4})$$

This equation, which does correctly describe the transformation law of the energy density of an electromagnetic wave, as in Eqn(10.14) above, has been obtained by combining the incorrect formula for the transformation of the energy density (E7.17) with the spurious length contraction effect in (E8.2). A factor $\gamma(1 - \beta \cos \phi)$ cancels to give the correct transformation law for a ‘light ray’ identified with a ‘light complex’.

The above calculation has defined a ‘light complex’ as a spherical region inside an infinite plane electromagnetic wave in the frame S, which is then transformed into the frame S'. However, from the Reciprocity Principle of special relativity [32] such a spherical ‘light complex’ can equally well be defined for the electromagnetic wave in the frame S'. Viewed from the frame S, by the same argument leading from (E8.1) to (E8.2), it will appear as an ellipsoid of revolution with volume V related to the volume of the sphere in S', V' by Eqn(8.3) with V and V' interchanged. The transformation law of the energy of this ‘light complex’ is then different to (E8.4) and inconsistent with that, (10.14), obtained from photon kinematics. The logical footing of this manifestly incorrect calculation is identical to that used by Einstein to derive (E8.4). That the latter describes correctly the transformation of the energy density of an electromagnetic wave is purely fortuitous.

Since the photon number density, n_γ , is Lorentz invariant, the relation (E8.4) is most simply obtained by transformation of unidimensional photon energy-momentum 4-vector:

$$p_\gamma \equiv (E_\gamma; (E_\gamma/c) \cos \phi, (E_\gamma/c) \sin \phi, 0) \quad (\text{11.1})$$

according to the space-time symmetric LT (10.3). After the derivation, by a fallacious argument, of the correct Eqn(E8.4), Einstein makes the following important remark:

It is remarkable that the energy and the frequency of a light complex vary with the state of motion of the observer in accordance with the same law.

In a paper written earlier in 1905 [13], Einstein had introduced into physics the concept of the ‘light quantum’ —light as a particle, later to be called the photon. However, at the time of writing the special relativity paper Einstein was not yet aware of the formalism of relativistic kinematics, to be developed somewhat later by Planck [47], and so could not have been known of the simple kinematical derivation of (E8.4) provided by the photon concept. Had he done so, he may have realised a deep and important connection between the light quantum and special relativity papers, that is, ultimately, one between quantum mechanics and special relativity. In fact, in the spirit of Einstein’s remark just quoted, the ratio of the frequency transformation equation:

$$\nu' = \nu \frac{1 - \beta \cos \phi}{\sqrt{1 - \beta^2}} \quad (\text{E7.13})$$

to the energy transformation equation (E8.4) implies the Lorentz invariance of E/ν , or that [14]:

$$\frac{E'}{\nu'} = \frac{E}{\nu} = \text{constant} \equiv h \quad (11.2)$$

This is the Planck-Einstein relation. If Einstein had had more confidence, in 1905, in the reality of photons the equations (E7.13) and (E8.4) of Ref. [2] might have lead to an independent discovery of Planck's constant and the associated fundamental quantum mechanical equation (11.2).

As discussed in Ref. [14], a more detailed comparison of plane electromagnetic waves with the equivalent photonic description enables other important quantum mechanical formulae and concepts to be understood as consistency conditions on the wave and particle representations of the same phenomenon.

The photonic description enables a simple and direct calculation of radiation pressure. The photon flux per unit area per unit time, f_γ , of a beam with angle of incidence ϕ on a mirror moving away from the beam with velocity v in the direction normal to its surface is:

$$f_\gamma = n_\gamma(c \cos \phi - v) \quad (11.3)$$

where n_γ is the photon number density. The momentum transfer, per photon, perpendicular to the surface of the mirror is:

$$\Delta p_\gamma = \frac{E_\gamma}{c} \cos \phi - \frac{E_\gamma'''}{c} \cos \phi''' \quad (11.4)$$

Use of (11.2) and (E8.14) enables this equation to be written as:

$$\Delta p_\gamma = \frac{2E_\gamma}{c} \frac{(\cos \phi - \beta)}{1 - \beta^2} \quad (11.5)$$

By Newton's Second Law the force per unit area perpendicular to the surface of the mirror, which is, by definition, the radiation pressure, is then:

$$P = f_\gamma \Delta p_\gamma = \frac{2E_\gamma n_\gamma (\cos \phi - \beta)^2}{1 - \beta^2} = \frac{2\rho_E (\cos \phi - \beta)^2}{1 - \beta^2} \quad (11.6)$$

With the replacement $\rho_E \rightarrow A^2/8\pi$ this agrees with Einstein's formula (E8.17).

Eqn(11.6) can be written in terms of the energy densities ρ_E and $\rho_{E'''}$ of the incident and reflected wave, respectively, as:

$$P = (\rho_E \cos \phi - \rho_{E'''} \cos \phi''')(\cos \phi - \beta) \quad (11.7)$$

Setting $\rho_E = A^2/8\pi$ and $\rho_{E'''} = (A''')^2/8\pi$ and using (E8.12) for A''' gives, for the radiation pressure:

$$P = \rho_E \frac{[2(1 - \beta^2) - (1 - 2\beta \cos \phi + \beta^2)(2\beta - (1 + \beta^2) \cos \phi)]}{(1 - \beta^2)^2} (\cos \phi - \beta) \quad (11.8)$$

in evident contradiction with (11.6).

To understand how Einstein nevertheless obtained the correct result (11.6) by using the incorrect energy density of the reflected wave, given by (E8.12), it is necessary to

examine in more detail the calculation presented. The energy flux per unit area per unit time, f_E , incident on the mirror is:

$$f_E = \frac{A^2(c \cos \phi - v)}{8\pi} \quad (\text{E8.15})$$

The derivation of this formula is demonstrated in Fig.7a. A portion of the plane wave (or, equivalently, of a monochromatic, parallel, photon beam) of dimensions $\Delta y = \Delta z = a$ is incident on the plane mirror moving with velocity v along the x -axis. It can be seen from the geometry of this figure that the number of photons crossing the mirror during the time interval t is:

$$N_\gamma^{IN} = n_\gamma a \text{Area}(\text{ABCD}) = n_\gamma a^2(c \cos \phi - v)t \quad (\text{11.9})$$

The incoming energy flux is therefore:

$$f_E = \frac{N_\gamma^{IN} E_\gamma}{a^2 t} = n_\gamma E_\gamma (c \cos \phi - v) = \frac{A^2(c \cos \phi - v)}{8\pi} \quad (\text{11.10})$$

since $n_\gamma E_\gamma = \rho_E = A^2/8\pi$, in agreement with (E8.15). The outgoing energy flux is given by Einstein as:

$$f_{E'''} = \frac{(A''')^2(-c \cos \phi''' + v)}{8\pi} \quad (\text{E8.16})$$

This formula is derived from the geometry of Fig.7b. However, the photon flux considered in this figure is evidently not that of the reflected beam (the number of photons reflected must be equal to the number of photons incident on the mirror) but rather is that of a beam of the same size and in the same direction as the reflected beam but *incident behind the mirror!* Indeed, it is found from the geometry of Fig.7b that:

$$N_\gamma^{OUT} = n_\gamma a \text{Area}(A'''B'''C'''D''') = n_\gamma a^2(-c \cos \phi''' + v)t \quad (\text{11.11})$$

which yields (E8.16) for the energy flux. The photon number density must be the same for the incident and reflected beams (the reflection process changes only the kinematical parameters of the photons, not their number). However, it is clear from (11.9) and (11.11) that in Einstein's calculation, $N_\gamma^{OUT} \neq N_\gamma^{IN}$. In Fig.3, $\phi = 30^\circ$, and $v = 0.25c$ so that $\phi''' = 132^\circ$. This gives:

$$\frac{N_\gamma^{OUT}}{N_\gamma^{IN}} = \frac{-\cos \phi''' + \beta}{\cos \phi - \beta} = 1.49 \quad (\text{11.12})$$

Therefore Einstein's reflected energy flux formula (E8.16) requires creation of photons in the reflection process, an evident absurdity.

Repeating Einstein's calculation of the radiation pressure by equating the net energy flow on to the mirror to the work done by the radiation pressure, but using the correct incident and reflected energy fluxes:

$$f_E^{IN} = n_\gamma E_\gamma (c \cos \phi - v) \quad (\text{11.13})$$

$$f_E^{OUT} = n_\gamma E_\gamma''' (c \cos \phi - v) = \frac{n_\gamma E_\gamma (1 - 2\beta \cos \phi + \beta^2)}{1 - \beta^2} (c \cos \phi - v) \quad (\text{11.14})$$

gives

$$P = \frac{cn_\gamma E_\gamma}{v} \left[1 - \frac{(1 - 2\beta \cos \phi + \beta^2)}{1 - \beta^2} \right] (\cos \phi - \beta) = 2 \left(\frac{A^2}{8\pi} \right) \frac{(\cos \phi - \beta)^2}{1 - \beta^2} \quad (11.15)$$

in agreement with (E8.17) and (11.5).

Finally, it must be concluded that Einstein obtained fortuitously the correct radiation pressure formula (E8.17) in spite of using the incorrect energy density formula (E7.17), and the absurd geometrical analysis of the reflected wave shown in (Fig.7b).

It remains that Einstein's final remark in this section is of great importance for the effective application of special relativity to physical problems.

All problems in the optics of moving bodies can be solved by the method here employed. What is essential is, that the electric and magnetic force of light which is influenced by a moving body be transformed into the system of co-ordinates at rest relatively to the body. By this means all problems in the optics of moving bodies will be reduced to a series of problems in the optics of stationary bodies.

The essential idea here, that physical problems are solved in the most elegant way by choosing the reference frame in which the description is simplest, and then using relativistic transformations to obtain predictions in the frame of interest, has applications extending far beyond 'the optics of moving bodies'. A related idea is to express physical equations, in any conveniently chosen frame, in terms of Lorentz-invariant quantities, to obtain an equation valid in all inertial reference frames [48, 8].

12 §9. Transformation of the Maxwell-Hertz Equations when Convection-Currents are Taken into Account

In this section the notation of Section 9 will be adopted, i.e. (x', y', z', t') will be used for space-time coordinates in S' rather than Einstein's (ξ, η, ζ, τ) . Also a field point in the xy -plane is considered where the source charge moves parallel to the x -axis. This choice of coordinates entails no loss of generality and much simplifies the equations since $\vec{E}_z = \vec{B}_x = \vec{B}_y = 0$. Also the substitution $\rho = 4\pi\rho_Q$ is made in Einstein's equations to obtain the conventional charge density ρ_Q in Gaussian units. In this case (E9.1)-(E9.11) simplify to:

$$\frac{1}{c} \frac{\partial E_x}{\partial t} + 4\pi\beta_u \rho_Q = \frac{\partial B_z}{\partial y} \quad (E9.1A)$$

$$\frac{1}{c} \frac{\partial E_y}{\partial t} = -\frac{\partial B_z}{\partial x} \quad (E9.2A)$$

$$\frac{1}{c} \frac{\partial B_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \quad (E9.3A)$$

$$4\pi\rho_Q = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \quad (\text{E9.4A})$$

$$\frac{1}{c} \frac{\partial E'_{x'}}{\partial t'} + 4\pi\beta_{u'}\rho'_Q = \frac{\partial B'_{z'}}{\partial y'} \quad (\text{E9.5A})$$

$$\frac{1}{c} \frac{\partial E'_{y'}}{\partial t'} = -\frac{\partial B'_{z'}}{\partial x'} \quad (\text{E9.6A})$$

$$\frac{1}{c} \frac{\partial B'_{z'}}{\partial t'} = \frac{\partial E'_{x'}}{\partial y'} - \frac{\partial E'_{y'}}{\partial x'} \quad (\text{E9.7A})$$

$$u' = \frac{u - v}{1 - uv/c^2} \quad (\text{E9.8A})$$

$$4\pi\rho'_Q = \frac{\partial E'_{x'}}{\partial x'} + \frac{\partial E'_{y'}}{\partial y'} = 4\pi\rho_Q\gamma(1 - \beta_u\beta) \quad (\text{E9.11A})$$

In these equations the force fields are specified at the space point $(x, y, 0)$. In general the source charge density ρ_Q is an arbitrary function of the coordinate (x_S, y_S, z_S) specifying an element of the source. For definiteness, this is specialised to the case of a single point-like source charge in uniform motion, where the charge density ρ_Q^* in the rest frame S^* of the charge is:

$$\rho_Q^*(\vec{x}_S) = Q\delta(\vec{x}_S - \vec{x}_Q) \quad (\text{12.1})$$

where \vec{x}_Q is the position vector of the source charge Q . In RCED and QED the force fields and the charge density are specified at the same time, so that the interchange force is instantaneous, not retarded [8].

Since the position of the field point and that of the source charge are always distinct there is in fact no difference between the ‘empty space’ case discussed in §6 and the ‘convection current included’ case of §9. Without proximate source charges or currents all force fields must vanish. The considerations of Section 9 above then imply that while the Faraday-Lenz law (E9.3A) and (E9.7A) and Ampère’s law for the transverse (to the direction of motion of the source charge) electric field, (E9.2A) and (E9.6A) are correct, the Ampère law equations for the longitudinal electric field, (E9.1A) and (E9.5A), and the Gauss law for the electric field, (E9.4A) and (E9.11A), are modified by covariance-breaking terms to:

$$\frac{1}{c} \frac{\partial E_x}{\partial t} + 4\pi J_x + \frac{Q\beta_u}{r^3} \left(\gamma_u - \frac{1}{\gamma_u} \right) (2 - 3\sin^2 \psi) = \frac{\partial B_z}{\partial y} \quad (\text{12.2})$$

$$4\pi J^0 + \frac{Q}{r^3} \left(\gamma_u - \frac{1}{\gamma_u} \right) (2 - 3\sin^2 \psi) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \quad (\text{12.3})$$

$$\frac{1}{c} \frac{\partial E'_{x'}}{\partial t'} + 4\pi J'_{x'} + \frac{Q\beta_{u'}}{r'^3} \left(\gamma_{u'} - \frac{1}{\gamma_{u'}} \right) (2 - 3\sin^2 \psi) = \frac{\partial B'_{z'}}{\partial y'} \quad (\text{12.4})$$

$$4\pi(J')^0 + \frac{Q}{r'^3} \left(\gamma_{u'} - \frac{1}{\gamma_{u'}} \right) (2 - 3\sin^2 \psi) = \frac{\partial E'_{x'}}{\partial x'} + \frac{\partial E'_{y'}}{\partial y'} \quad (\text{12.5})$$

where the transformation law of the charge density in the last member of (E9.11A) as applied between the frames S^* and S has been used to introduce the 4-vector current density:

$$(J^0; \vec{J}) \equiv (\rho_Q; \beta_u\rho_Q, 0, 0) = (\gamma_u\rho_Q^*; \gamma_u\beta_u\rho_Q^*, 0, 0) \quad (\text{12.6})$$

Given that the ‘empty space’ Maxwell equations of §6 already describe the electric and magnetic force fields at ‘source free’ field points in an identical manner to the ‘with source’ equations of the present section it is perhaps worthwhile to ask what additional physics information is provided by the latter set of equations? There are two answers to this question, the first concerning predictions for the force fields and the second the description of radiative processes where real photons are created.

The operational meaning of the force fields is provided by the Lorentz equation (9.1) in terms of the force on a test charge. The Maxwell equations contain spatial and temporal partial derivatives of these fields. What additional information is provided by these first-order partial differential equations? The answer, insofar as the force fields are concerned, is not at all clear. In fact predictions of electromagnetic forces are typically extracted from Maxwell’s equations by invoking the corresponding integral relations, which, in a certain sense, annul the partial derivatives:

Gauss’ Law for the electric field

$$\int_S \vec{E} \cdot d\vec{S} = 4\pi \int_V \rho_Q^* dV \quad (12.7)$$

The magnetostatic Ampère law

$$\int_s \vec{B} \cdot d\vec{s} = 4\pi \int_S \vec{J} \cdot d\vec{S} \quad (12.8)$$

The Faraday-Lenz law

$$\int_s \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S} \quad (12.9)$$

In highly symmetrical configurations these integral relations provide a simple manner to calculate the fields and hence, by use of the Lorentz equation, the physical prediction, which is the force on a test charge. For example Coulomb’s inverse square force law for a point charge may be derived from (12.7) [49] and the magnetic field due to an infinite straight conductor from (12.8). Eqn(12.9) gives directly the induction force on conduction electrons in a circular coil. However, all the interchange forces of electrodynamics are described by the single formula (3.4) in which no ‘fields’ appear and no Lorentz equation must be invoked. Maxwell’s equations, including their covariance-breaking relativistic corrections, are all found *en route* to the derivation of Eqn(3.4) [8, 11] (see Fig.1). Electric and magnetic force fields, although of tremendous historical importance and phenomenological utility, do not provide the most economical and fundamental description of electromagnetic forces. This is given by quantum electrodynamics, in which the underlying quantum mechanical mechanism is the exchange between electric charges of space-like virtual photons [8].

The second application of Maxwell’s equations with sources concerns an entirely different process: the production and propagation of real photons, associated, in classical electrodynamics, with accelerated electric charges. In common with essentially all text book authors Einstein did not distinguish between force fields (effects of virtual photon exchange) and radiation fields (creation propagation and destruction of real photons) using

identical symbols for the different types of fields and referring in the text to the ‘force’ of the radiation fields discussed in §7 and §8. This is not the place to describe in detail how Maxwell’s equations with sources are used to calculate radiation fluxes. This is done in all advanced text books on classical electromagnetism. Only the essential steps are reviewed, and in view of the incorrect transformation law of the energy density of an electromagnetic wave (E7.17) given by Einstein the (lack of) covariance of the ‘radiation fields’ will be commented on

The electric and magnetic fields in the Gauss and Ampère laws (E9.4A) and (E9.1A) are replaced by the derivatives of the 4-vector potential A^α according to the defining equations (9.2) and (9.3) to yield second order partial differential equations in components of A^α . The Lorenz condition:

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial A^0}{\partial t} = 0 \quad (12.10)$$

is then used to obtain separate second-order partial differential equations (d’Alembert Equations) for \vec{A} and A^0 . Solving these equations by the Green’s function technique and calculating the corresponding retarded electric and magnetic fields, using (9.2) and (9.3), terms with both $1/r^2$ and $1/r$ dependence are obtained. Terms of the second type, associated with source charge acceleration, are identified as radiation fields. Making the non-relativistic approximation that the maximum source-charge velocity is much less than c , and using the Poynting vector to calculate the energy flux, the rate of real photon radiation in, for example, dipole radiation is predicted, in good agreement both with experiment and, in the appropriate limit with the corresponding quantum mechanical calculation [11]. At very large distances from the source, the radiation field, denoted by Einstein as A , has the same operational meaning as the probability amplitude to observe a photon at the corresponding spatial position [11, 14]. The physical interpretation of the radiation field is therefore quite different from that of force fields which are quite unrelated to the real photon degrees of freedom.

It is interesting to note that the Lorenz condition (12.10) is an identity for the instantaneous force-field 4-vector potential, being a consequence of the relation (9.24) connecting temporal and spatial partial derivatives [8]. Nevertheless its introduction is essential for the derivation of the retarded potential associated with the radiation fields,

It was found above in Section 10 that the force fields of Eqns(E7.1)-(E7.3) and (E7.5)-(E7.7) naively identified as radiation fields gave the incorrect transformation laws (E7.17),(E7.18) for the energy density of a plane electromagnetic wave. Another example of the inadequacy of classical electromagnetic fields to give a correct relativistic description of real photon processes is discussed in Ref. [11]. Although radiation fields derived from Maxwell’s equations, as outlined above, give a correct description of dipole radiation in the non-relativistic limit of source charge motion, this is no longer true if the fields are boosted from the frame in which the average velocity of the source charge vanishes, into a frame with $\beta_u \simeq 1$. The energy flow predicted by the Poynting vector of the transformed radiation fields does then not agree with that obtained by direct Lorentz transformation of the energy-momentum 4-vectors of the radiated photons.

Einstein closes §9 with the following remark:

In addition I may briefly remark that the following important law may be

easily deduced from the developed equations: If an electrically charged body is in motion anywhere in space without altering its charge when regarded from a system of co-ordinates moving with the body, its charge also remains --when regarded from the ‘‘stationary’’ system S-- constant.

This statement is true insofar as the value of the charge Q in the 4-vector electromagnetic current (12.6) is a Lorentz-invariant quantity. However the ‘effective charge’, proportional to the force on a test charge at a fixed relative position, does vary with the motion of the source. Inspection of (9.21) shows that the longitudinal electric field is reduced by the fraction $\beta_u^2/2$ at lowest order in β_u , while the transverse field is increased by the same fraction. On averaging over an isotropic distribution of directions for a source of fixed velocity the radial electric field is increased, at leading order in β_u , by the fraction $\beta_u^2/6$.

A consequence of this is that a test charge close to a magnetostatic system experiences an electric field $\propto I^2$ where I is the magnet current. This is because the effective charge of moving conduction electrons is different to that of the static positive ions of the conductor material. The static transverse field $\propto I^2$ of a superconducting magnet was observed and published [50] in 1976, in agreement with the prediction of the RCED formula (9.21). The same transverse electric field is predicted by the CEM formula (9.19) when $\psi = \pi/2$.

It is important to notice that this ‘electrostatic field in magnetostatics’ has no effect on the experiments which have tested, to very high precision, the neutrality of matter by subjecting it to strong external electric fields and searching for the effect of electric forces [51, 52]. This is because, unlike the dependence of electric field produced by a charge on the motion of the charge, the force on a test charge due to an external electric field does not depend on the motion of the charge. See the Lorentz force equation (9.1).

13 §10. Dynamics of the Slowly Accelerated Electron

In this section, the motion in different reference frames, of a particle of charge e under the influence of electric and magnetic fields is considered. The particle is assumed to be instantaneously at rest at the origin of S' at the instant at which the origins of S and S' coincide. Only the electric force acts in the frame S' to give the equations of motion (Newton’s Second Law):

$$m \frac{d^2 \xi}{d\tau^2} = e E'_\xi \quad (\text{E10.4})$$

$$m \frac{d^2 \eta}{d\tau^2} = e E'_\eta \quad (\text{E10.5})$$

$$m \frac{d^2 \zeta}{d\tau^2} = e E'_\zeta \quad (\text{E10.6})$$

Using the transformation of space-time coordinates and field components:

$$\xi = \gamma(x - vt), \quad \eta = y, \quad \zeta = z, \quad \tau = \gamma(t - vx/c^2) \quad (\text{E10.7})$$

$$E'_\xi = E_x, \quad E'_\eta = \gamma(E_y - \beta B_z), \quad E'_\zeta = \gamma(E_z + \beta B_y) \quad (\text{E10.8})$$

the following equations of motion in the frame S are obtained:

$$m\gamma^3 \frac{d^2x}{dt^2} = eE_x = eE'_\xi \quad (\text{E10.12})$$

$$m\gamma^2 \frac{d^2y}{dt^2} = e\gamma(E_y - \beta B_z) = eE'_\eta \quad (\text{E10.13})$$

$$m\gamma^2 \frac{d^2z}{dt^2} = e\gamma(E_z + \beta B_y) = eE'_\zeta \quad (\text{E10.14})$$

The derivation of (E10.12)-(E10.14) from (E10.4)-(E10.8) is discussed below, but in order to compare these predictions with those of the RCED formulae, a specific source of the fields, a charge Q moving with fixed velocity u along the x -axis in S, as in Sections 9 and 12, will be considered. The same notation for space time-coordinates and field components as in these sections will also now be employed. Since, in this case, $E_z = B_x = B_y = 0$ Eqns(E10.6) and (E10.14) are no longer relevant. Eqns(E10.4) and (E10.5) are written as:

$$m \frac{d^2x'}{dt'^2} = eE'_{x'} \quad (\text{E10.4A})$$

$$m \frac{d^2y'}{dt'^2} = eE'_{y'} \quad (\text{E10.4B})$$

and Eqns(E10.12) and (E10.13) as:

$$m\gamma^3 \frac{d^2x}{dt^2} = eE_x = eE'_{x'} \quad (\text{E10.12A})$$

$$m\gamma^2 \frac{d^2y}{dt^2} = e\gamma(E_y - \beta B_z) = eE'_{y'} \quad (\text{E10.13A})$$

Using the RCED force fields of (9.21) and (9.22), the Lorentz force equation (9.1), and the field transformation laws discussed in Section 9, the following equations of motion are found in the frame S:

$$\frac{dp_x}{dt} = \frac{d(\gamma\beta_x mc)}{dt} = m\gamma^3 \frac{d^2x}{dt^2} = eE_x = e \frac{\gamma_w E'_{x'}}{\gamma_u} \quad (\text{13.1})$$

$$\frac{dp_y}{dt} = \frac{d(\gamma\beta_y mc)}{dt} = m\gamma \frac{d^2y}{dt^2} = e(E_y - \beta B_z) = e \frac{E'_{y'}}{\gamma} \quad (\text{13.2})$$

where the source charge moves with velocity w along the x' -axis in S'. To derive the third member of Eqn(13.2) from the second the condition $\beta_y = 0$ has been used. Agreement is found between the first member of Eqn(E10.12A) and the third member of (13.1), however the last member of (E10.12A) is at variance that of (13.1) and, although (E10.13A) and (13.2) are algebraically equivalent, the forces defined by these equations differ by a multiplicative factor γ .

As will be shown below the relation:

$$\frac{\text{transverse force in test charge rest frame}}{\text{transverse force in frame with relative } x\text{-velocity } v} = \gamma \quad (\text{13.3})$$

satisfied by (13.2) but not by (E10.13A), is of general validity, following from the invariance of transverse coordinates with respect to the LT.

Since the methods used to derive, on the one hand (E10.12A) and (E10.13A), and on the other (13.1) and (13.2), are different it is perhaps not surprising that different results are obtained. Einstein introduces as a *hypothesis* the equality of the electromagnetic forces in the frames S' and S, and simply rewrites the S' frame fields in terms of the S frame ones according to the transformation equations (E10.8). In the case of the RCED equations (13.1) and (13.2), the fields are directly calculated from the source currents using (9.21) and (9.22) for the frame S and (9.28) and (9.29) for the frame S' and substituted into the Lorentz force equation (9.1). The left-hand sides of (13.1) and (13.2) are just an expression of the relativistic version of Newton's Second Law. The forces calculated in this way are found not to be equal in the frames S' and S, in contradiction to Einstein's assumption in Eqns(E10.12A) and (E10.13A).

In order to derive the left sides of (E10.12A) and (E10.13A), the coordinates x' , y' and t' of (E10.4A) and (E10.5A) are transformed using the LT of (E10.7) with the replacements $\xi \rightarrow x'$, $\eta \rightarrow y'$ and $\tau \rightarrow t'$. No details were provided of the calculation, so it must be conjectured how it was performed. A calculation based on a certain assumption will now be shown to yield the left sides of (E10.12A) and (E10.13A) from those of (E10.4A) and (E10.5A). The assumption is that v and γ in the LT in (E10.7) may be considered constant under the conditions of the problem. This is clearly in the spirit of the title of §10: 'Dynamics of the Slowly Accelerated Electron' and the supposed 'slow' motion of the electron, the latter meaning, presumably, $v \ll c$. However $v \equiv dx/dt$ (and hence v and γ) cannot be constant, as assumed, as this implies that $d^2x/dt^2 = 0$. Making, in any case, this assumption, and considering infinitesimal space and time intervals, the ratio of the first to the fourth equation in (E10.7) gives:

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2} \frac{dx}{dt}} \quad (13.4)$$

The ratio of the second to the fourth equations of (E7.10) gives

$$\frac{dy'}{dt'} = \frac{\frac{dy}{dt}}{\gamma(1 - \frac{v}{c^2} \frac{dx}{dt})} \quad (13.5)$$

The fourth equation of (E10.7), written with infinitesimal intervals, yields the differential operator relation:

$$\frac{d}{dt'} = \frac{1}{\gamma(1 - \frac{v}{c^2} \frac{dx}{dt})} \frac{d}{dt} \quad (13.6)$$

Using (13.6) in combination with (13.4) or (13.5) and assuming that v (but not dx/dt) is constant yields the equations:

$$\frac{d^2x'}{dt'^2} = \frac{1}{\gamma(1 - \frac{v}{c^2} \frac{dx}{dt})} \left[\frac{1}{1 - \frac{v}{c^2} \frac{dx}{dt}} + \frac{v(\frac{dx}{dt} - v)}{c^2(1 - \frac{v}{c^2} \frac{dx}{dt})^2} \right] \frac{d^2x}{dt^2} \quad (13.7)$$

$$\frac{d^2y'}{dt'^2} = \frac{1}{\gamma^2(1 - \frac{v}{c^2} \frac{dx}{dt})} \left[\frac{1}{1 - \frac{v}{c^2} \frac{dx}{dt}} + \frac{v \frac{dy}{dt}}{c^2(1 - \frac{v}{c^2} \frac{dx}{dt})^2} \right] \frac{d^2y}{dt^2} \quad (13.8)$$

Setting finally $dx/dt = v$ and $dy/dt = 0$ (the test charge moves along the x -axis in S) in (13.7) and (13.8) give:

$$\frac{d^2x'}{dt'^2} = \gamma^3 \frac{d^2x}{dt^2}, \quad \frac{d^2y'}{dt'^2} = \gamma^2 \frac{d^2x}{dt^2} \quad (13.9)$$

relating the left sides of (E10.12A),(E10.13A) to those (E10.4A),(E10.5A).

Two comments are in order:

- (i) The velocity v is not constant under the conditions of the problem.
- (ii) Eqn(13.6) does not correctly describe the relation between the t and t' derivatives.

To demonstrate (ii) consider the differential form of the LT in (E10.7):

$$\frac{dx'}{dt} = \gamma(t) \left[\frac{dx}{dt} - v(t) \right] \quad (13.10)$$

$$\frac{dt'}{dt} = \gamma(t) \left[1 - \frac{v(t)}{c^2} \frac{dx}{dt} \right] \quad (13.11)$$

Where the time dependence (non-constant values) of v and γ is explicitly indicated. Since, by hypothesis, the test charge is at rest at the origin of S': $x' = 0$, $dx'/dt = 0$ and $dx/dt = v(t)$, so that (13.11) may be written as:

$$dt = \gamma(t) dt'. \quad (13.12)$$

describing an instantaneous time dilatation effect. The correct relation between the time derivatives is therefore not (13.6) but instead:

$$\frac{d}{dt'} = \gamma(t) \frac{d}{dt} \quad (13.13)$$

Since, as discussed in Section 9, space intervals are Lorentz invariant, $dx = dx'$, (13.13) gives:

$$\frac{d^2 x'}{dt'^2} = \gamma(t) \frac{d}{dt} \left[\gamma(t) \frac{dx}{dt} \right] = \gamma(t)^4 \frac{d^2 x}{dt^2} \quad (13.14)$$

Since $dy = dy'$, it follows similarly that;

$$\frac{d^2 y'}{dt'^2} = \gamma(t)^4 \frac{d^2 y}{dt^2} \quad (13.15)$$

These equations should be contrasted with Einstein's results in (13.9). Correcting (E10.12A) by replacing the factor γ^3 on the left by γ^4 it can be seen that this equation is no longer in agreement with Eqn(13.1). Also the factor γ^2 on the left side of (E10.13A) should be replaced by γ^4 . This equation of transverse motion will be discussed further below.

The evident difference between the corrected equations (E10.12A) and (E10.13A) and the RCED equations of motion (13.1) and (13.2) should come as no surprise. The physical assumptions underlying these equations are quite different. In (E10.12A) and (E10.13A) the forces in S and S' are, by hypothesis, equal. Simply, the electric fields in S' have been transformed into the fields in S according to the transformation laws in (E10.8). Similarly the derivatives on the left sides are transformed (actually incorrectly), according to the space-time transformations of (E10.7). But there is no physics in such a purely mathematical transformation. In the case of (13.1) and (13.2) the electromagnetic forces are calculated using the appropriate formula in each frame, and are found not be equal. The

left sides of the equations contain no transformations, but are expressions of the relativistic generalisation of Newton's Second Law —a 'force', in any frame, may be operationally defined as the time derivative of the momentum in that frame. The agreement between (E10.12A) and (13.1) as a consequence of the incorrect second derivative transformation equation in (13.9) must then be regarded as purely fortuitous.

The general relation (13.3), not respected by (E10.13A), will now be proved. It follows from the invariance of the transverse coordinate interval under the LT:

$$dy = dy' \quad (13.16)$$

and the time dilatation relation (13.12). Eqn(13.16) implies conservation of relativistic transverse momentum:

$$p'_T = m \frac{dy'}{dt'} = m \frac{dy}{dt} = \gamma m \frac{dy}{dt} = p_T \quad (13.17)$$

Since the transverse force is defined, in any frame, as dp_T/dt and (13.17) states that transverse momentum is conserved, $dp_T = dp'_T$,

$$F_T \equiv \frac{dp_T}{dt} = \frac{dp'_T}{\gamma dt'} \equiv \frac{F'_T}{\gamma} \quad (13.18)$$

which implies

$$\frac{F'_T}{F_T} = \gamma \quad (13.19)$$

which is Eqn(13.3)

It is important to remark that if Einstein had consistently applied the condition that the electron is 'slow' in the frame S, i.e. $v \ll c$, as well a similar condition on the velocity of the source charge $u \ll c$, so that terms of $O(\beta^2)$, $O(\beta_u^2)$ and $O(\beta\beta_u)$ and higher orders are neglected, Eqns(13.1), (E10.12A) and (13.2),(E10.13A) are identical to first order in β :

$$m \frac{d^2x}{dt^2} = eE_x = eE_{x'} + O(\beta^2, \beta_u^2) \quad (13.20)$$

$$m \frac{d^2y}{dt^2} = e(E_y - \beta B_z) = eE'_{y'} + O(\beta^2) \quad (13.21)$$

So, at this order, Einstein has indeed derived the Lorentz force equation as a consequence of the transformation of the electric field in the rest frame of the test charge, as previously described in §6. In this context it is interesting to recall Einstein's remark in §6, concerning the equality of the forces on a test charge in frames in which it is either in motion or at rest. The important caveat there mentioned: 'which, if we neglect the terms multiplied by the second and higher powers of v/c ', is forgotten in the last members of (E10.12A) and (E10.13A) where the forces in the frames S and S' are instead assumed to be exactly equal, in equations where some terms of 'the second and higher powers of v/c ' have been retained

The RCED equations (13.1) and (13.2) are valid for all velocities, not only in the small β limit. It is crucial for Einstein's subsequent arguments that the first member of (E10.13A), although derived on the basis of the false assumptions pointed out in the

remarks (i) and (ii) above, is actually identical to the third member of (13.1) and so is relativistically correct, that is, valid to all orders in β . This is essential for the derivation of the seminal equation (E10.17) where Einstein first introduces the ‘ $E = mc^2$ ’ concept.

Invoking the Newtonian definition: Force = mass \times acceleration, the concepts of ‘longitudinal’ and ‘transverse’ masses are suggested by the left sides of Eqns(E10.12A) and (E10.13A) respectively:

$$\text{Longitudinal mass} = \frac{m}{(\sqrt{1 - \beta^2})^3} \quad (\text{E10.15})$$

$$\text{Transverse mass} = \frac{m}{1 - \beta^2} \quad (\text{E10.16})$$

Although the definition of ‘longitudinal mass’ is the same as that given by Lorentz in the previous year [25] the ‘transverse mass’ is $\propto \gamma^2$ to be compared with Lorentz’ definition where it is $\propto \gamma$. Thus Lorentz’ definitions of both masses are consistent with the RCED formulae (13.1) and (13.2). The difference between the Lorentz and Einstein definitions of transverse mass has been described in the literature as a different ‘choice of convention in the expression for force and mass in the dynamics of charged particles.’ [53]. This is misleading. The ‘convention’ happens to be correct for the longitudinal mass but not for the transverse one. Stated more bluntly, Eqns(E10.13A) and (E10.16) are both wrong since the mathematically illicit calculation by which they are derived does not, unlike in the case of (E10.12A) and (E10.15), give, fortuitously, the correct result.

In the English translation of Ref. [2] a footnote mentioning the work of Planck [47] states that the Newtonian definition of force as mass times acceleration is perhaps not the one best adapted to special relativity, but rather one in which ‘the laws momentum and energy assume the simplest form’. This prompts another ‘What if?’ speculation similar to the one in Section 11 above, concerning the Planck-Einstein relation (11.2). What if Einstein had adopted instead the definition of a force (suggested by Newton’s First Law), as the time derivative of the momentum, as in Eqns(13.1) and (13.2)? Having obtained the correct equation in the first member of (E10.12A) this would suggest the following definition for the momentum of the electron:

$$\begin{aligned} p &= \int eE_x dt = m \int \gamma^3 \frac{d^2x}{dt^2} dt = mc \int \gamma^3 \frac{d\beta}{dt} dt \\ &= mc \int_0^\beta \frac{d\beta}{(1 - \beta^2)^{\frac{3}{2}}} = m\gamma\beta c \end{aligned} \quad (13.22)$$

This is just the relativistic momentum, introduced later by Planck [47].

After obtaining the equations of motion in the frame S, (E10.12)-(E10.14), and introducing Longitudinal and Transverse masses, the following comment is made:

We remark that these results as to the mass are also valid for ponderable material points, because a ponderable material point can be made into an electron (in our sense of the word) by the addition of an electric charge *no matter how small*.

(Italics in the original) This is interesting, because, from a modern viewpoint, it is understood that the mass variation is a purely kinematical effect, valid for both charged and neutral particles independently of any dynamics. Einstein derived the formulae for charged particles in the context of classical electrodynamics and it is not clear from this statement that dynamical and purely kinematical consequences of special relativity were clearly separated in his mind at this point.

The next paragraph concerns the most important result of Ref. [2], both as a revolutionary physical concept and for its practical consequences. The kinetic energy, W , of the electron is calculated by the space integral of the x -component of the force in the frame S using the first member of Eqn(E10.12) or (E10.12A):

$$W = \int eE_x dx = m \int_0^v \gamma^3 v dv = mc^2 \left\{ \frac{1}{\sqrt{1-\beta^2}} - 1 \right\} \quad (\text{E10.17})$$

There follows the statement:

Thus when $v = c$, W becomes infinite. Velocities greater than that of light have --as in our previous results-- no possibility of existence.

This has become a paradigm of 20th Century physics, often termed ‘causality’. It is true of all physical objects with a time-like or light-like energy-momentum 4-vector such that

$$E^2 - p^2 c^2 \geq 0 \quad (13.23)$$

It is not, however, true for virtual photons that, according to QED, are responsible for inter-charge electromagnetic forces. These have a space-like energy-momentum 4-vector with

$$E^2 - p^2 c^2 < 0 \quad (13.24)$$

The speed of such particles is always greater than that of light and is predicted by QED to be infinite in the overall centre-of-mass frame of the interacting charges [8].

Einstein did not state in Ref. [2] the equivalence of mass and energy that is implicit in (E10.17). Later in 1905 he published another paper with the title ‘Does the Inertia of a Body Depend upon its Energy Content’ [46] in which an ingenious thought experiment involving (in modern terms) the emission of a pair of photons, as viewed from two different inertial frames, was analysed, to draw the conclusion:

If a body gives off energy L in the form of radiation its mass diminishes by L/c^2 .

The analysis of the thought experiment was based on the previously found and correct (but incorrectly derived) formula (E10.4) for the transformation of the energy of a plane electromagnetic wave. At this stage light quanta were still ‘heuristic’ and the equations of relativistic particle kinematics, later to be written down by Planck, were not available.

However, it is already clear from (E10.17) that the term mc^2 has the dimensions of energy and is associated with the mass of the particle. This equation may be rewritten as

$$E = W + mc^2 \quad (13.25)$$

where

$$E \equiv \frac{mc^2}{\sqrt{1-\beta^2}} = \gamma mc^2 \quad (13.26)$$

If Einstein had taken the time integral of (E10.12) to obtain the momentum, Eqn(13.22) would have been obtained. This equation, (13.26) and the identity $\gamma^2 \equiv \gamma^2\beta^2 + 1$ gives

$$E^2 = p^2c^2 + m^2c^4 \quad (13.27)$$

while the ratio of (13.22) to (13.26) gives

$$v = \frac{pc^2}{E} \quad (13.28)$$

Combining (13.27) and (13.28):

$$E(v=0) \equiv E_0 = mc^2 \quad (13.29)$$

Thus the equations of relativistic kinematics (13.25)-(13.29) were already implicit in the formulae given in Ref. [2], so that no further thought experiment was required to obtain Eqn(13.29).

Finally, after noting that, by energy conservation, a change in potential energy equal to the kinetic energy W in (E10.12) is obtained (Eqn(E10.19)), the motion of an electron in a constant magnetic field was considered, making use of (E10.13). The magnetic field is parallel to the z -axis, but as throughout the paper, the source remains unspecified. The equation of motion obtained from (E10.13) is:

$$-\frac{d^2y}{dx^2} = \frac{v^2}{R} = \frac{e}{m}\beta B_z\sqrt{1-\beta^2} \quad (E10.20)$$

which yields, for the radius of curvature, R , of the circular orbit:

$$R = \frac{mc^2}{e} \frac{\beta}{\sqrt{1-\beta^2}} \frac{1}{B_z} \quad (E10.21)$$

It is informative to compare these results with the prediction of Eqn(13.2). Placing a source charge of opposite sign, at rest in S , at the same position as the moving source charge, will produce no additional magnetic field, and according to Eqn(9.21) will reduce the value of the electric field in (13.2) from E_y to $(1 - 1/\gamma_u)E_y$. This opposite charge corresponds, in the present problem, to the effect of the positive charge of the ions of the bulk matter in a conductor used to produced a magnetic field. The non-vanishing electric field is just the ‘electrostatics in magnetostatics’ effect discussed in the previous section. Taking it into account gives, for the instantaneous radius of curvature, \mathcal{R} , of the electron orbit:

$$\mathcal{R} = \frac{\gamma mc^2}{e} \frac{\beta^2}{[\beta B_z - E_y(1 - 1/\gamma_u)]} \quad (13.30)$$

The right side of (E10.21) is recovered in the limit $\beta_u \rightarrow 0$.

14 Summary: Einstein's Mistakes

As stated in the Introduction, and implicit in the title of Ref. [2], the purpose of Einstein's formulation of Special Relativity Theory (SRT)¹⁴, on the basis of the Special Relativity Principle and the constancy of the speed of light, was:

...the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies.

This reliance on the phenomena of Classical Electromagnetism (CEM) in constructing the theory is manifest in the 'Kinematical Part' of Ref. [2], through the adoption of the Light Signal Clock Synchronisation Procedure (LSCSP) to *define* simultaneous and spatially separated events. Since SRT describes the physics of (flat) space-time that underlies *all* dynamical physical phenomena (with the possible exception of gravity), Einstein's approach is, *a priori*, a perfectly valid one. However, applying Occam's razor, it is not the simplest and most economical one (i.e. that with the smallest number of, and the simplest, necessary initial postulates) to obtain SRT. This was already realised as early as 1910 by Ignatowsky [30]. The space-time Lorentz Transformation (LT), from which all purely kinematical consequences of SRT may be obtained, does not require, for its derivation, the consideration of SRT or of any other dynamical theory [54, 1, 32, 33]. The derivations, of the latter type, of the LT, in Refs. [32, 33], shown in Fig.4, may be compared with Einstein's (putative) derivation shown in Fig.3.

After establishing the LSCSP in §1, Einstein discusses in §2 a thought experiment which purports to demonstrate, without invoking the LT, 'relativity of simultaneity' (RS) by applying the LSCSP of §2 to two moving clocks at different positions in the same inertial frame. The analysis presented of this thought experiment is Einstein's first major mistake. The LSCSP is defined for clocks at rest in some inertial frame; it makes no sense to apply the same equations to moving clocks, the velocities of which, relative to light signals, in the frame under consideration, are not equal to c . The LSCSP is clearly based on the assumption of a constant velocity of light signals relative to the to-be-synchronised clocks, which are at rest in the frame in which the procedure is applied. As shown in Fig.5 and Table 1, the events considered by Einstein in the 'stationary' frame S are not the Lorentz-transformed LSCSP events in the 'moving' frame S'. The explicit calculation presented in Section 5 shows that the clocks introduced by Einstein at A and B, which have an apparent (time dilated) rate equal to that of clocks at rest in S, are judged to be synchronous by observers in both S and S', in contradiction to Einstein's conclusion.

In the derivation of the LT in §3, the false correspondence of the events in S in Fig.5c and 5e with those in S' in Fig.5b and 5d respectively, assumed in §2, is maintained. As described in detail in Section 6 above, that the correct LT for a synchronised clock at the origin of S' (E3.27)-(E3.30) is obtained from the incorrect assignment of events in S and S' just mentioned, is due to other, compensating, errors in the derivation. The first is the neglect of the x -dependence of the LT in the initial ansatz for the time transformation

¹⁴For the reader's convenience, some acronyms introduced earlier in the paper are redefined in the present section

equation: $\tau = \tau(x', y, z, t)$ instead of $\tau = \tau(x', x, y, z, t)$ where $x' = L = x(t = 0)$ is a clock synchronisation constant. The second is the neglect of this constant in the LT finally obtained. In the derivation, the space time coordinates in the final equations obtained are actually those of events at the point B, whereas the LT derived is appropriate only to a clock at A (the origin in S'). Einstein's 'derivation' of the LT is therefore fallacious, both with respect to the initial assumptions concerning the transformed events, and with respect to the correct physical interpretation of the space-time coordinates appearing in the final result.

The second major mistake in Ref. [2] is the 'length contraction' (LC) effect of Eqn(E4.2) resulting from the failure to take into account the appropriate value of x' (or L) for the clocks concerned. The LT for a synchronised clock at the center of the considered sphere, at rest in S', is applied also to points on the surface of the sphere, where it does not correctly describe synchronised clocks. Use of the correct LT for a point on the surface of the sphere shows (Eqn(7.8)) that there is no apparent distortion of the sphere due to the LT when it is viewed from S. Interestingly enough, the standard RS effect (7.9) associated with the spurious LC effect of (E4.2), as described in all text books on SRT, is not mentioned in Ref. [2]. The RS effect of §2, obtained without invoking the LT, although equally spurious, is related to different events in S and S', than those giving (7.9).

Einstein's discussion of asymmetric aging, by considering observations of a stationary clock and a moving one, after the correct derivation of the Time Dilatation (TD) formula (E4.4), is incomplete, since only an observer in the 'stationary' frame S is considered. Similar asymmetric aging (but in the opposite sense) will be seen by an observer at rest in the 'moving' frame S' when clocks at rest in S are compared with those at rest in S' in the experiment reciprocal to the one considered. Einstein did not comment on this apparently paradoxical situation in Ref. [2].

As pointed out in Refs. [38, 39, 40], Einstein's discussion of the relative rates of clocks at a pole of the Earth and the equator did not take into account gravitational (general relativistic) effects that, to a high degree of accuracy, render equal the rates of clocks at sea level at any position on the Earth's surface.

Turning now to the 'Electrodynamic Part' of Ref. [2], it should be pointed out that the transformation laws of electric and magnetic fields, (E9.13)-(E6.15), obtained by imposing the covariance of Maxwell's equations in 'free space' under the LT, are based on the tacit assumption that electric and magnetic fields are local 'classical' ones for which the space and time coordinates may be taken as independent variables in the differential calculus. Recalling the operational meaning of electromagnetic fields as proportional to the force on a test charge at the corresponding space-time position, it is clear that such fields cannot exist in 'free space' —there must be at least one source charge in the proximity of the field point to produce the fields. Taking this fact into account, the simplest non-trivial electrodynamical system, where mechanical forces act, must consist of at least two charges, one the 'test charge' and the other the 'source charge'. When such a system is considered it is clear from the formulae giving the fields of a uniformly moving charge, in either CEM or the recently developed relativistic theory (RCED) [8], that, in the case that the source charge moves along the x -axis, the derivatives with respect to time and the x -coordinate are not independent, as required for a local classical field, but are related by Eqn(9.24).

This formula has been previously given in text books on CEM [55, 56] but its implication for the transformation laws of electric and magnetic fields has only recently been worked out [11]. Because time and the x -coordinate are not independent variables for the force fields of both CEM and RCED, these fields do not necessarily transform as a second-rank tensor according to the equations (9.7) and (9.15)-(9.18) or Eqns(E6.13)-(E6.15). Explicit calculation of the transformation laws of the RCED fields between the frames S and S' using Eqns(9.21)-(9.22) and (9.28)-(9.29) shows agreement with the tensor transformation law for the transverse electric and magnetic fields given by Eqns(E6.14) and (E6.15), but not for the longitudinal component of the electric field. The first equation in (E6.13):

$$E'_{x'} = E_x \quad (14.1)$$

is replaced by the first formula in (9.30):

$$E'_{x'} = \frac{\gamma_u}{\gamma_w} E_x \quad (14.2)$$

where the source charge moves with velocities u, w along the x -axis in the frames S and S' respectively. Thus the longitudinal electric field, unlike the transverse electric and magnetic fields, does not transform in a covariant manner —its value depends on the source charge velocities in both S and S'. In fact there is, in the problem, a ‘preferred frame’ that breaks special relativistic covariance —it is the frame, S*, where the source charge is at rest.

It is also shown by direct calculation of spatial and temporal derivatives using the RCED fields (9.21) and (9.22), and the relation (9.24), that Ampère’s Law is a necessary consequence of the electric field Gauss Law, thus deriving Maxwell’s ‘Displacement Current’ term in the former, and that both the x -component of Ampère’s Law and the electric field Gauss Law are modified by covariance-breaking terms that depend on the velocity of the source charge. Einstein’s initial hypothesis for the derivation of Eqs(E6.13)-(E6.15) —the covariance of Maxwell’s Equations— is therefore invalid for the RCED force fields. However, the y - and z -components of Ampère’s Law, the Faraday-Lenz Law and the magnetic Gauss Law remain covariant for the RCED force fields.

At the end of §6, Einstein re-discusses the problem, introduced in the Introduction, of electromagnetic induction in different reference frames, in the light of the field transformation laws (E6.13)-(E6.15). Already in the Introduction, Einstein indicates that the ‘luminiferous aether’ of the 19th Century will become superfluous; here it is further suggested that the ‘electromotive force’ (the magnetic Lorentz force) and, by implication, the magnetic field itself, is only an ‘auxiliary concept’. By the same token it could be argued that the electric field, also defined by the force on a test charge, is a similar ‘auxiliary concept’, but Einstein does not take this further step. Somewhat inconsistently, Einstein is prepared to throw away the aether, but not the ‘electromagnetic waves’ supposed throughout the 19th Century and beyond, to propagate in it. It is tantamount to throwing away the ocean but not the waves on the shore. What was needed to complete Einstein’s revolution was the concept of light as a particle, that replaces, in an ontological sense, the ‘electromagnetic waves’. In spite, however, of having won the Nobel Prize for the discovery of the photon it is not clear that Einstein, even to the end of his life¹⁵, ever

¹⁵In a letter to Besso in 1951, Einstein made the comment on the subject of the light quantum ‘Heute glaubt zwar jeder Lump, er wisse es, aber er träucht sich’[57].

embraced the concept, trivially accepted by all practitioners of experimental High Energy Physics, that the photon is a elementary particle like any other such. From a modern viewpoint (see Ref. [8]) both electric and magnetic fields are second level mathematical abstractions. The true seat of the fundamental underlying physics is to be found in QED processes. For the mechanical effects described, classically, by electric and magnetic force fields, it is the exchange, in a symmetric manner, of space-like virtual photons between the source and test charges.

Einstein's failure to obtain the noncovariant transformation law (14.2) of the longitudinal electric field results from the introduction of such fields as the *a priori* physical concepts in terms of which the theory is framed, without properly taking into account the nature of the physical phenomenon (inter-charge forces) that is to be described with the aid of the fields. When this is done it is quite clear that the fields are not classical and local but have an essential functional dependence on the physical parameters of the source charge (or charges) both for the case of retarded and instantaneous inter-charge interactions. Einstein, also, in common with the authors of many text books on CEM makes no distinction (using identical mathematical symbols) between the 'force fields' considered in §4, §9 and §10 and the 'radiation fields' considered in §7 and §8, although the operational meaning of the fields is quite different in each case.

In §7, devoted to the Doppler effect for electromagnetic waves and the aberration of light, Einstein presumably derives the associated formulae by assuming equality of the phases defined by Eqns(E7.4) and (E7.8), Lorentz transforming the space-time coordinates in (E7.8) into the S frame using the LT (E3.27)-(E3.30) and equating the coefficients of the S frame coordinates to obtain the correct relations (E7.9)-(E7.15). However, as explained in Section 10, such a procedure is of somewhat dubious mathematical validity. Both phases actually vanish, due to the vanishing of the factor in curly brackets, in each case. The phases are therefore equal (and zero) for arbitrary finite values of ω and ω' as well as those satisfying Eqn(E7.9). As shown in Section 10, a less dubious derivation of (E7.9)-(E7.15) is provided by exploiting the manifestly Lorentz-scalar property of the phase.

At the end of §7, the formulae (E7.17) and (E7.18) for the transformation law of the square of the amplitude of what Einstein calls 'the amplitude of the electric or magnetic force' are written down, without any derivation. In view of the similarity between the numerators and denominators of the right sides of (E7.13) and (E7.17) it may be conjectured that Einstein is confusing the concepts of frequency and field amplitude (or incorrectly assuming that they are proportional) at this point. At the beginning of the following §8 it is stated that:

Since $A^2/8\pi$ equals the energy of light per unit volume we have to regard $(A')^2/8\pi$, by the principle of relativity, as the energy of light (presumably also per unit volume) in the moving system.

This statement correctly identifies A and A' as the field amplitudes of a plane electromagnetic wave in the frames S and S' respectively. They are not 'the amplitude of the electric or magnetic force' as stated in the previous section. Indeed this last definition is physically meaningless, since A and A' correspond to radiation fields that describe, classically, a parallel beam of monochromatic photons [14], not intercharge forces. This

demonstrates Einstein's confusion concerning the different physical interpretations that must be assigned to force fields and radiation fields. Since $A^2/8\pi = \rho_E$ where ρ_E is the energy density of the plane electromagnetic wave, Eqns(E7.17) and (E7.18) give, according to Einstein, the transformation law of this energy density. They are incorrect. This is the third major error in Ref. [2]. The correct transformation law for ρ_E , Eqn(10.11), is actually given by the later equation (E8.4), on making the replacements $E \rightarrow \rho_E$, $E' \rightarrow \rho'_E$. The quantities E and E' in Eqn(E8.4) are the total energies of what Einstein calls a 'light complex'. In order to derive the correct transformation for ρ_E from the incorrect transformation law for A^2 , (E7.17), the 'light complex' is defined as an arbitrary spherical region, in the frame S, of an (infinite) plane electromagnetic wave, and the spurious LC effect derived in §4 is applied. The volume of the resulting ellipsoid of revolution, supposedly observed in the frame S', to that of the sphere in S is the reciprocal of the ratio of A' to A given by Eqn(E7.17). Consideration of the total energy of the contracted 'light complex' in S' then leads to the formula (E8.4), which must actually replace the incorrect formula (E7.17) as the transformation law for ρ_E or A^2 . As previously, in the derivation of the LT, incorrect formulae, (E7.17) and (E8.2) are combined in such a way as to give a correct result, here the transformation law (E8.4), on replacing the spurious quantity, E: 'energy of the light complex', by $A^2/8\pi$ or ρ_E . Indeed, in view of the frequency transformation formula (E7.13), the Planck-Einstein relation and the Lorentz invariance of the photon number density, the correct transformation law of ρ_E , Eqn(10.11), is an immediate consequence of (E7.13).

In the discussion of the radiation pressure due to reflection of a plane electromagnetic wave at oblique incidence on a uniformly moving plane mirror in the second part of §8, the incorrect formula (E7.17) for the transformation of the amplitude of an electromagnetic wave is used, yielding the incorrect formulae (E8.6) and (E8.12) used in the analysis of the reflection problem. The formula finally obtained for the light pressure (E8.17) is correct, but in order to derive it using the wrong amplitude transformation equations (E8.6) and (E8.12) Einstein commits the fourth (and most flagrant) major mistake in Ref. [2]. The formula (E8.16), that is purported to be the energy flux of the reflected wave is actually the energy flux of a wave of the same dimensions and direction as the reflected wave but *incident on the back surface of the mirror!* (see Fig.7b). The writer of the present paper finds it very difficult to understand why, to his best knowledge, this obvious blunder has not been pointed out in the century since Ref. [2] was written. Combining this absurd flux calculation with the incorrect formula (E8.12) for A''' , gives, fortuitously, the correct result (E8.17) for the radiation pressure. In fact the flux of reflected photons is equal to the flux of incident photons (in the reflection process exactly one photon is created for each photon that is destroyed). The different energy flux then results solely from the transformation of photon energy by reflection. The correct result for the radiation pressure is then obtained by using (E8.4) or (10.11) to transform ρ_E and assuming equal fluxes of incident and reflected photons. This calculation is in Eqns(11.3)-(11.6). Alternatively an 'energy flow' analysis, as done by Einstein, can be performed, using the correct photon fluxes and energy densities to obtain the same result. This calculation yields Eqns(11.13)-(11.15).

The fifth and sixth major mistakes in Ref [2] occur in the concluding section §10, 'Dynamics of the Slowly Accelerated Electron', but as in previous cases discussed above, the effects of the errors cancel in the final, and in this case crucial, result. Thus the

formula (E10.12), that is integrated to obtain the seminal relation (E10.7) demonstrating the equivalence of mass and energy, is correct.

The fifth error is a conceptual one. In order to derive the differential equation of motion of the ‘Slowly Accelerated Electron’ in the frame S from that in the frame S’ (where it is instantaneously at rest), the coordinates and fields in the former equation are simply transformed into those of the frame S’ using the equations (E10.7)-(E10.8). Such a procedure correctly predicts the Lorentz Force Equation in the frame S to the lowest order in β (i.e. neglecting corrections of $O(\beta^2)$). However, since only a change of variables is performed, it is assumed that the force components are equal in the frames S and S’. Explicit calculation of the forces for the case of a uniformly moving source charge using the RCED equations (9.21) and (9.22) shows that the forces are not equal in the two frames when $O(\beta^2)$ and higher order corrections are included. Einstein’s initial hypothesis in deriving Eqns(E10.12)-(E10.14) is therefore not correct in the relativistic theory.

The sixth error occurs in the calculation of the Jacobian relating $d^2\xi/d\tau^2$ in Eqn(10.4) to d^2x/dt^2 in Eqn(10.12). This calculation assumes that v and γ are constant in Eqn(10.7), but that dx/dt is not. Since $v \equiv dx/dt$, this is impossible. Calculating correctly the Jacobian, allowing for the time dependence of v and γ , and the Lorentz invariance of spatial intervals $d\xi = dx$, results in the replacement $\gamma^3 \rightarrow \gamma^4$ in the left side of (10.12) and $\gamma^2 \rightarrow \gamma^4$ in the left sides of (E10.13) and (E10.14). The spatial integration of the so-modified Eqn(E10.12) then no longer yields the mass-energy equivalence relation (E10.17). In fact the γ^3 factor on the left side of (E10.12) is correct, but it originates not from the Jacobian of a coordinate transformation but from the *definition of force as the time derivative of momentum* and the definition, (13.32), of relativistic momentum (see Eqn(13.1)).

To conclude this section a list of the incorrect formulae from Ref. [2] is given, together with, in each case, the corresponding correct relativistic formula.

(i) Space-time Lorentz Transformation

$$\tau = \gamma(t - \frac{vx}{c^2}) \quad (\text{E3.27})$$

$$\xi = \gamma(x - vt) \quad (\text{E3.28})$$

$$\eta = y \quad (\text{E3.29})$$

$$\zeta = z \quad (\text{E3.30})$$

The equations (E3.27) and (E3.28) are correct only for a synchronised clock at $\xi = 0$. For such a clock at $\xi = L$ they are replaced by:

$$\tau = \gamma[t - \frac{v(x - L)}{c^2}] \quad (14.3)$$

$$\xi - L = \gamma[x - L - vt] = 0 \quad (14.4)$$

or, equivalently:

$$t = \gamma\tau \quad (14.5)$$

$$x = L + vt \quad (14.6)$$

(ii) Spurious Length Contraction Effect

$$\frac{x^2}{1 - (v/c)^2} + y^2 + z^2 = R^2 \quad (\text{E4.2})$$

is replaced by

$$x^2 + y^2 + z^2 = R^2 \quad (\text{14.7})$$

(iii) Transformation of the Energy Density of Plane *EM* Wave

$$(A')^2 = \frac{A^2(1 - \beta \cos \phi)^2}{1 - \beta^2} \quad (\text{E7.17})$$

$$(A')^2 = \frac{A^2(1 - \beta)}{1 + \beta} \quad (\text{E7.18})$$

are replaced by:

$$(A')^2 = \frac{A^2(1 - \beta \cos \phi)}{\sqrt{1 - \beta^2}} \quad (\text{14.8})$$

$$(A')^2 = A^2 \sqrt{\frac{1 - \beta}{1 + \beta}} \quad (\text{14.9})$$

Also E'/E in Eqns(E8.4), (E8.5) is replaced by $(A')^2/A^2$.

$$A' = A \frac{1 - \beta \cos \phi}{\sqrt{1 - \beta^2}} \quad (\text{E8.6})$$

$$A''' = A'' \frac{1 + \beta \cos \phi''}{\sqrt{1 - \beta^2}} = A \frac{(1 - 2\beta \cos \phi + \beta^2)}{1 - \beta^2} \quad (\text{E8.12})$$

are replaced by:

$$(A')^2 = A^2 \frac{1 - \beta \cos \phi}{\sqrt{1 - \beta^2}} \quad (\text{14.10})$$

$$(A''')^2 = (A'')^2 \frac{1 + \beta \cos \phi''}{\sqrt{1 - \beta^2}} = A^2 \frac{(1 - 2\beta \cos \phi + \beta^2)}{1 - \beta^2} \quad (\text{14.11})$$

The reflected energy flux:

$$f_{E'''} = \frac{(A''')^2(-c \cos \phi''' + v)}{8\pi} \quad (\text{E8.16})$$

is replaced by:

$$f_{E'''} = \frac{(A''')^2(c \cos \phi - v)}{8\pi} = f_E (A''')^2 / A^2 \quad (\text{14.12})$$

where f_E is the incident energy flux.

(iv) Ampère's Law and the Gauss Law for the Electric Field

$$\frac{1}{c} \frac{\partial E_x}{\partial t} + 4\pi \beta_u \rho_Q = \frac{\partial B_z}{\partial y} \quad (\text{E9.1A})$$

$$4\pi \rho_Q = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \quad (\text{E9.4A})$$

$$\frac{1}{c} \frac{\partial E'_{x'}}{\partial t'} + 4\pi \beta_{u'} \rho'_Q = \frac{\partial B'_{z'}}{\partial y'} \quad (\text{E9.5A})$$

$$4\pi\rho'_Q = \frac{\partial E'_{x'}}{\partial x'} + \frac{\partial E'_{y'}}{\partial y'} \quad (\text{E9.11A})$$

are replaced by

$$\frac{1}{c} \frac{\partial E_x}{\partial t} + 4\pi\beta_u\rho_Q + \frac{Q\beta_u}{r^3} \left(\gamma_u - \frac{1}{\gamma_u} \right) (2 - 3\sin^2\psi) = \frac{\partial B_z}{\partial y} \quad (14.13)$$

$$4\pi\rho_Q + \frac{Q}{r^3} \left(\gamma_u - \frac{1}{\gamma_u} \right) (2 - 3\sin^2\psi) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \quad (14.14)$$

$$\frac{1}{c} \frac{\partial E'_{x'}}{\partial t'} + 4\pi\beta_{u'}\rho'_Q + \frac{Q\beta_{u'}}{r^3} \left(\gamma_{u'} - \frac{1}{\gamma_{u'}} \right) (2 - 3\sin^2\psi) = \frac{\partial B'_{z'}}{\partial y'} \quad (14.15)$$

$$4\pi\rho'_Q + \frac{Q}{r^3} \left(\gamma_{u'} - \frac{1}{\gamma_{u'}} \right) (2 - 3\sin^2\psi) = \frac{\partial E'_{x'}}{\partial x'} + \frac{\partial E'_{y'}}{\partial y'} \quad (14.16)$$

(v) Differential Equation of Motion of Accelerated Electron

$$m\gamma^3 \frac{d^2x}{dt^2} = eE_x = eE'_\xi \quad (\text{E10.12})$$

$$m\gamma^2 \frac{d^2y}{dt^2} = e\gamma(E_y - \beta B_z) = eE'_\eta \quad (\text{E10.13})$$

$$m\gamma^2 \frac{d^2z}{dt^2} = e\gamma(E_z + \beta B_y) = eE'_\zeta \quad (\text{E10.14})$$

are replaced by

$$\frac{dp_x}{dt} = \frac{d(\gamma\beta_x mc)}{dt} = m\gamma^3 \frac{d^2x}{dt^2} = eE_x = e \frac{\gamma_w E'_{x'}}{\gamma_u} \quad (14.17)$$

$$\frac{dp_y}{dt} = \frac{d(\gamma\beta_y mc)}{dt} = m\gamma \frac{d^2y}{dt^2} = e(E_y - \beta B_z) = e \frac{E'_{y'}}{\gamma} \quad (14.18)$$

$$\frac{dp_z}{dt} = \frac{d(\gamma\beta_z mc)}{dt} = m\gamma \frac{d^2z}{dt^2} = e(E_z + \beta B_y) = e \frac{E'_{z'}}{\gamma} \quad (14.19)$$

$$\text{Transverse mass} = \frac{m}{1 - \beta^2} \quad (\text{E10.16})$$

is replaced by

$$\text{Transverse mass} = \frac{m}{\sqrt{1 - \beta^2}} \quad (14.20)$$

15 Conclusions . The Enduring Legacy of Einstein's 1905 Special Relativity Paper

To complement the list of wrong formulae presented at the end of the previous section the present one starts with a list of the correct formulae from Ref. [2], leaving aside the question whether the derivations given were physically valid or, as discussed above, in

some cases, fallacious. In conclusion, the importance and originality of the relativistic predictions of Ref. [2] will be discussed. In all equations in this section, the notation for times and spatial coordinates introduced in Section 2 will be employed.

(a) The Lorentz transformation for a clock at the origin of S' and the time dilatation effect

$$t' = \gamma[\tau - \frac{vx}{c^2}] \quad (15.1)$$

$$x' = \gamma[x - v\tau] = 0 \quad (15.2)$$

$$y' = y \quad (15.3)$$

$$z' = z \quad (15.4)$$

(15.1) and (15.2) may also be written as:

$$\tau = \gamma t' \quad (15.5)$$

$$x = v\tau \quad (15.6)$$

where (15.5) describes the time dilatation effect, giving the apparent time, t' , of a clock at rest in S', as viewed from S, and τ is the corresponding time registered by a similar and synchronised clock at rest in S.

(b) Velocity addition formulae

$$V = \frac{\sqrt{v^2 + \omega^2 + 2v\omega \cos \alpha - [(v\omega \sin \alpha)/c^2]^2}}{1 + \frac{v\omega}{c^2} \cos \alpha} \quad (15.7)$$

or, when $\alpha = 0$ (parallel velocity addition)

$$V = \frac{v + \omega}{1 + \frac{v\omega}{c^2}} \quad (15.8)$$

See also the formulae for x - y - and z -components in (h) below.

(c) Transformation laws for transverse electric fields and magnetic fields

$$E'_{y'} = \gamma[E_y - \beta B_z] \quad (15.9)$$

$$E'_{z'} = \gamma[E_z + \beta B_y] \quad (15.10)$$

$$B'_{y'} = \gamma[B_y + \beta E_z] \quad (15.11)$$

$$B'_{z'} = \gamma[B_z - \beta E_y] \quad (15.12)$$

(d) Doppler shift and aberration of light

$$\nu' = \nu\gamma(1 - \beta \cos \phi) \quad (15.13)$$

or, when $\phi = 0$

$$\nu' = \nu \sqrt{\frac{1 - \beta}{1 + \beta}} \quad (15.14)$$

$$\cos \phi' = \frac{\cos \phi - \beta}{1 - \beta \cos \phi} \quad (15.15)$$

- (e) Transformation of the energy density, ρ_E , of a plane electromagnetic wave

$$\rho'_{E'} = \rho_E \gamma (1 - \beta \cos \phi) \quad (15.16)$$

or, when $\phi = 0$,

$$\rho'_{E'} = \rho_E \sqrt{\frac{1 - \beta}{1 + \beta}} \quad (15.17)$$

These formulae are obtained by making the substitutions $E \rightarrow A^2/(8\pi) = \rho_E$, $E' \rightarrow (A')^2/(8\pi) = \rho'_{E'}$ in the transformation formulae (E8.4) and (E8.5) of Einstein's 'light complex'. They are identical to the transformation laws of the energy of a photon given by the replacements $\rho_E \rightarrow E_\gamma$, $\rho'_{E'} \rightarrow E'_\gamma$

- (f) Radiation pressure of a plane electromagnetic wave incident on a moving mirror

$$P = 2 \frac{A^2}{8\pi} \frac{(\cos \phi - \beta)^2}{1 - \beta^2} \quad (15.18)$$

- (g) Transformation of the y -component of Ampère's Law and of the Faraday-Lenz Law

Here is assumed, as in Section 12, that the source charge moves with velocity v along the x -axis in the frame S , and that the field point lies in the x - y plane

In S :

$$\frac{1}{c} \frac{\partial E_y}{\partial t} = -\frac{\partial B_z}{\partial x} \quad (15.19)$$

$$\frac{1}{c} \frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} \quad (15.20)$$

In S' :

$$\frac{1}{c} \frac{\partial E'_{y'}}{\partial t'} = -\frac{\partial B'_{z'}}{\partial x'} \quad (15.21)$$

$$\frac{1}{c} \frac{\partial \vec{B}'}{\partial t'} = -\vec{\nabla}' \times \vec{E}' \quad (15.22)$$

- (h) Transformation of the charge and current densities

In S :

$$(c\rho_Q; \vec{u}\rho_Q) = (c\rho_Q; u_x\rho_Q, u_y\rho_Q, u_z\rho_Q) \quad (15.23)$$

In S' :

$$(c\rho'_Q; \vec{u}'\rho'_Q) = (c\rho'_Q; u'_{x'}\rho'_Q, u'_{y'}\rho'_Q, u'_{z'}\rho'_Q) \quad (15.24)$$

where:

$$u'_{x'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \quad (15.25)$$

$$u'_{y'} = \frac{u_y}{\gamma(1 - \frac{u_x v}{c^2})} \quad (15.26)$$

$$u'_{z'} = \frac{u_z}{\gamma(1 - \frac{u_x v}{c^2})} \quad (15.27)$$

$$\rho'_Q = \rho_Q \gamma (1 - \frac{u_x v}{c^2}) \quad (15.28)$$

(i) Charge motion in electric and magnetic fields

If the charge, e , of mass m moves instantaneously parallel to the x -axis then:

$$m \frac{d^2 x}{dt^2} = eE_x + O(\beta^2) \quad (15.29)$$

$$m \frac{d^2 y}{dt^2} = e[E_y - \beta B_z] + O(\beta^2) \quad (15.30)$$

$$m \frac{d^2 z}{dt^2} = e[E_z + \beta B_y] + O(\beta^2) \quad (15.31)$$

or, equivalently in 3-vector notation:

$$m \frac{d^2 \vec{x}}{dt^2} = e[\vec{E} + \vec{\beta} \times \vec{B}] + O(\beta^2) \quad (15.32)$$

To all orders in β :

$$\frac{dp_x}{dt} = m\gamma^3 \frac{d^2 x}{dt^2} = eE_x \quad (15.33)$$

(j) The equivalence of mass and energy

$$W \equiv \text{kinetic energy} = (\gamma - 1)mc^2 \quad (15.34)$$

or, equivalently,

$$E \equiv \text{relativistic energy} \equiv \gamma mc^2 = mc^2 + W \quad (15.35)$$

When the object is at rest, $W = 0$, and $E \rightarrow E_0$ where

$$E_0 \equiv \text{relativistic rest energy} = mc^2 \quad (15.36)$$

(k) Motion of a charge in the plane perpendicular to a magnetic field \vec{B}

$$R = \text{radius of orbit} = \frac{mc^2}{e} \frac{\beta}{\sqrt{1 - \beta^2}} \frac{1}{B} \quad (15.37)$$

In judging the value of any scientific work which satisfies the necessary pre-requisites of internal self-consistency and formal correctness, two questions are paramount: firstly, is it original? and secondly, is it important? These criteria will first be applied, point-by-point, to the list, (a)-(k), of correct results from Ref. [2] just given, before, finally, discussing Einstein's methodology in more general terms, and giving a personal opinion about the merit of the work presented in Ref. [2] from the perspective of the 21st Century.

Concerning (a), the LT in the form (15.1)-(15.4) was first given by Larmor in 1900 [24, 27]. The transformation given by Lorentz in 1904 in his last pre-relativity paper [25] was identical to that of Larmor [24, 27]. The transformation was given the name 'Lorentz Transformation' by Poincaré in 1905 [26]. The essential difference from earlier, or contemporary but independent, authors was that Einstein claimed for the first time to *derive* the LT from some simple axioms. Although, as shown in Section 6 above, the 'derivation' of Ref. [2] was actually fallacious, the essential idea was taken up by many later authors, so that, at the present time, the literature devoted to the derivation of the

LT on the basis of various initial postulates is vast. See Refs. [31, 32] for citations of only the derivations that do not employ Einstein's second postulate.

The time dilatation prediction (15.5) in (a) is one of the two key predictions of Ref. [2] that changed, for ever, a fundamental conception of physics. As a physical consequence of the relativistic transformation of time it was first pointed out by Larmor in 1897 [60, 27] who noted that orbiting electrons in a uniformly moving inertial frame will appear, to a stationary observer, to be moving slow by the factor $1/\gamma$, or $1 - \beta^2/2$ at $O(\beta^2)$. In differential form, (15.5) is the basis for relativistic kinematics. The energy-momentum 4-vector, P , in the frame S , of a physical object of mass m , that is at rest in the frame S' , is derived from the space-time 4-vector $X \equiv (c\tau; \vec{x})$ in S according to the relation:

$$P \equiv m \frac{dX}{dt'} = m\gamma \frac{dX}{d\tau} = m(\gamma c; \gamma \vec{v}) \equiv mV \quad (15.38)$$

where V is the 4-vector velocity in the frame S . The differential form $d\tau = \gamma dt'$ of the time dilatation formula (15.5) is essential for this definition of P and the consequent formulae of relativistic kinematics.

Although Lorentz had previously introduced the concept of 'local time' as early as 1895 [61], and the differential form of (15.5) appears as an intermediate step (among many others) in the section of Poincaré's 1906 relativity paper [62] entitled 'Contraction of Electrons', it seems reasonable to claim that Einstein was the first to publish the interpretation of the TD relation (15.5) as a universal property of the observed time of *any* uniformly moving clock.

The velocity addition formulae of (b) and (h) above were first given, independently, by Einstein in Ref. [2] and Poincaré [62].

The transformation laws (15.9)-(15.12) of transverse electric fields and magnetic fields in (c) were first published by Larmor in 1900 [24, 27]. They were also given, independently of Einstein's work, by Lorentz in 1904 [25], and Poincaré in 1906 [62].

To the present writer's best knowledge, the formulae in (d) for the relativistic Doppler shift and aberration of light were given for the first time by Einstein in Ref. [2]. They contain implicitly, in view of the Planck-Einstein relation $E_\gamma = h\nu$, the relativistic kinematics of photons. Similarly, given the photon concept, the transformation formulae for the energy density of a plane electromagnetic wave, (15.16) and (15.17) in (e) also follow directly from (15.13) and the Planck-Einstein relation [14].

The correct relativistic light pressure formula, (15.8), in (f) was obtained for the first time by Einstein in Ref. [2], albeit as the result of an absurdly incorrect derivation (see Section 11 above).

The covariance of the transverse components of Ampère's Law and of the Faraday-Lenz Law in (g) were independently demonstrated by Larmor in 1900 [24, 27], Lorentz in 1904 [25], and by Poincaré in 1906 [62].

The transformation of the charge and current densities in (15.23)-(15.28) of (h) were also independently obtained by Poincaré [62]. It is interesting to note that the 4-vector character of the charge and current densities in (15.23) and (15.24), also pointed out

by Poincaré [62], as well the 4-vector character of $U \equiv (\gamma_u c; \gamma_u \vec{u})$, are also implicit in Einstein's formulae (15.23)-(15.28). Choosing \vec{u} parallel to the x -axis, (15.24), (15.25) and (15.28) give:

$$\rho'_Q = \gamma(\rho_Q - \frac{(\rho_Q u)v}{c^2}) \quad (15.39)$$

$$\rho'_Q u' = \gamma(\rho_Q u - v \rho_Q) \quad (15.40)$$

Comparing (15.39) and (15.40) with (15.1) and (15.2), it can be seen that ρ_Q and $\rho_Q u$ transform in the same manner as τ and x respectively under the LT, and so constitute a 4-vector. Using the relation (15.28) to transform the charge density in the rest frame of the source charge, ρ_Q^* into the frames S and S' gives the relations:

$$\rho_Q = \gamma_u \rho_Q^* \quad (15.41)$$

$$\rho'_Q = \gamma_{u'} \rho_Q^* \quad (15.42)$$

Substitution of (15.41) and (15.42) into (15.39) and (15.40) gives:

$$\gamma_{u'} = \gamma(\gamma_u - \frac{(\gamma_u u)v}{c^2}) \quad (15.43)$$

$$\gamma_{u'} u' = \gamma(\gamma_u u - v \gamma_u) \quad (15.44)$$

showing that γ_u and $\gamma_u u$ also transform as τ and x , establishing the 4-vector character (already manifest for $V \equiv (\gamma c; \gamma \vec{v})$ in (15.38)) for $U \equiv (\gamma_u c; \gamma_u u, 0, 0)$.

In (i) the right side of (15.32) gives the Lorentz force, the magnetic part of which also, as explained in §6: ‘...which *if we neglect the terms multiplied by second and higher powers of v/c* (my italics) is equal to the vector product of the velocity of the charge and the magnetic force’ (should be ‘magnetic field’) ‘divided by the velocity of light’. In fact, the right side of (15.32) gives the Lorentz force correctly to all orders in v/c ; the left side, however, is the time derivative of the momentum, as in Eqns(14.17)-(14.19), not, as in Einstein's formulae ‘mass \times acceleration’. This elegant derivation of the Lorentz force equation, directly from the transformation laws of electric and magnetic fields, although only valid to first order in β , gives a deep insight into what Einstein calls the ‘auxiliary’ nature of the ‘magnetic field’ concept relative to the physical effect —the force on a test charge— considered. It may be contrasted with Lorentz' derivation of his force formula based on a Lagrangian containing the electromagnetic potential and the Principle of Least Action. This, also discussed in detail by Poincaré [62], is by comparison, lengthy and rich in mathematical detail, but weak in physical insight. A similar derivation may be found in the well-known text-book of Landau and Lifschitz [63] as well as in Ref.[8] by the present author.

Eqn(15.33) gives the correct relativistic generalisation of Newton's Second Law, for the problem considered. The last member of this equation was independently given by Poincaré in Section 7 of Ref. [62]. However neither Einstein nor Poincaré introduced the concept of relativistic momentum implicit in the first member of (15.33). The second member of the equation is crucial for the derivation of the mass-energy equivalence relation (15.34) of (j). The most original and, in view of practical applications, most important, prediction of Ref. [2] is that of (j) —the equivalence of mass and energy. For

this it is necessary to introduce the definition of relativistic energy of (15.35) by mathematical substitution in (15.34). The resulting formula, as well as the ‘ $E = mc^2$ ’ relation (15.36) are both of completely general validity. The paper, published by Einstein later in 1905 [13], containing the (controversial¹⁶) thought experiment relating (to the lowest order in β^2 only) the change of mass of an object with radiated electromagnetic energy is then unnecessary to establish the mass-energy equivalence. Although Poincaré had noted as early as 1900 that the momentum of an electromagnetic wave is $1/c^2$ times the energy flux given by the Poynting vector [67], suggesting, in view of the classical definition of momentum, a mass density $1/c^2$ times the energy density, Einstein’s equation (15.34) shows clearly the general equivalence of mass and energy as well as demonstrating that c is the limiting speed of any object that has a time-like energy momentum 4-vector.

Einstein’s relation for the radius of curvature of the orbit of an electron in a uniform magnetic field was obtained previously by Lorentz [25] and even demonstrated by him in the same paper to be in reasonable agreement with measurements of Kaufmann dating from 1902 [64]. This means that some experimental evidence for the non-Newtonian mechanics of SRT existed even before SRT was invented! However other predictions based, like that of Lorentz, on specific electrodynamical models of the electron due to Abraham and Bucherer could not be excluded in 1904. It was not until 1908 that the SRT prediction of (15.36) was experimentally verified by Bucherer [65].

Einstein is now universally considered to be the founding father of SRT. Is this a fair assesment in view of the previously published work of Larmor and Lorentz, and the contemporary, but independent, work of Poincaré? It is sometimes claimed that it was Einstein who introduced the Special Relativity Principle into physics and that this principle was not (or only partially) understood by the other authors just cited. The present writer finds it very difficult to understand this, since this Principle was clearly stated by Galileo in a graphic manner [66] and was also well known to Newton. Two lines of algebra suffice to demonstrate the invariance of Newton’s Second Law under Galilean transformations. This Principle—that the laws of physics are the same in all inertial frames—is true in both classical Newtonian mechanics and in relativistic mechanics. It is sometimes claimed that Einstein was unique in extending the Special Relativity Principle, known to hold in classical mechanics, to the ‘rest of physics’, in particular to electrodynamics. However, from a modern viewpoint, many aspects of the latter are most easily understood in terms of the relativistic kinematics of real or virtual photons [14, 8, 11]. Thus, at the fundamental level, there is no real distinction between electrodynamics and either classical (but relativistic) mechanics and quantum mechanics.

One important difference between Einstein, on the one hand, and Larmor, Lorentz and Poincaré on the other, was that the latter authors were all working on aether theories or electrodynamic models of electron structure whereas Einstein clearly stated that the aether was ‘superfluous’ in his approach. However Einstein still clung to the concept of classical fields, and of wave motion in such fields, whereas in all other domains of physics such fields require some material support for their existence. The modern solution, after rejecting the aether, is to embrace the photon concept that renders also ‘superfluous’ electric and magnetic force fields [8] and identifies radiation fields as the classical limit

¹⁶In 1952 Ives claimed that the argument in Ref. [13] was circular [58]. Ives’ arguments were contested by later authors. See Ref. [59] and references therein.

of quantum mechanical probability amplitudes [14, 11]. Einstein never did this, either in Ref. [2] or in any later work.

Pro-Einstein advocates often state, with some justification, that he was the first to understand that SRT deals with space-time geometry and kinematics, not with dynamics. This seems not to be borne out by titles of the papers in which the related work was published: Larmor, ‘On a dynamical theory of the electric and luminiferous medium’ [60] and ‘Aether and Matter’ [24]; Lorentz, [25] ‘Electromagnetic phenomena in a system moving with any velocity less than that of light’, Einstein, [2] ‘On the electrodynamics of moving bodies’; Poincaré [62] ‘On the dynamics of the electron’. The only paper that does not specifically mention dynamics is that of Lorentz. Given that the ‘moving body’ introduced in the mechanical section §10 of Ref. [2] is an ‘electron’ the titles of the Einstein and Poincaré papers are very similar. However large parts of the latter are an attempt to describe electrons, considered from a modern viewpoint as just one type of elementary particle among many others, as a structure generated by the electromagnetic field and other forces of unspecified origin. This subject was not considered by Einstein. However, Einstein’s caveat concerning the physical meaning of the differential equations of motion and the longitudinal and transverse masses of his ‘electron’ indicates that he did not fully appreciate, at the time of writing Ref. [2], the purely kinematical nature of these formulae.

Looking at the list, (a)-(h), of formally correct predictions, only the formulae for Doppler shift and aberration, the radiation pressure formula and the mass-energy equivalence relation are to be found in Einstein’s work and not that of either Larmor and Lorentz, and/or Poincaré. In particular, the predictions in (a), (b), (c), (g), (h) and (j) were also all given (at least as formulae) by Poincaré in Ref.[62].

Some writers on the history of SRT have tended to be unfair to either Poincaré [68] or to Einstein [69, 70], while they, and many others, have largely ignored the work of Larmor. A notable exception is to be found in Pauli’s monograph on SRT written in 1921 and re-issued in English translation in 1958 [54]. In particular, Larmor’s priority for the LT is pointed out. Also the space-time geometric aspects of SRT, usually associated with the work of Minkowski [71], as well as the Group Theory of the LT, both of which were first developed by Poincaré in Ref. [62], are cited by Pauli.

The major difference between the work of Einstein and that of his contemporaries lies, however, not in the mathematical results obtained but rather in style and methodology—the introduction of thought experiments, not only, as previously in popular scientific, but also in research, literature, and concentration on essential points using the minimal amount of mathematics to resolve the physical problems addressed. In common with Newton is the use of the minimum number of, and simplest possible, initial postulates, from which the conclusions are deduced by mathematical logic¹⁷. Einstein saw very clearly that the space-time LT, if it was not to remain a mathematical abstraction, had to describe real, physical, clocks. This important concept is to be found nowhere in work of Larmor and Lorentz, or (in contrast to his popular essays) in the research papers of Poincaré. Such clocks were an essential part of the thought experiments discussed in §1, §2, §3 and §4 of Ref. [2]. Unfortunately, as pointed out in Sections 5, 6 and 7 above, both the light signal clock synchronisation procedure of §2 and the physical meaning of the second

¹⁷This point was particularly stressed by Holton [53]

postulate defined in this section are misinterpreted. Crucially, the additive constants ‘to be placed on the right side of each of these (LT) equations’ were also neglected, leading to a fallacious ‘derivation’ of the LT and spurious LC and RS effects, although only the LC effect was discussed in Ref. [2], the RS effect discussed there, though also of a spurious nature (see Section 5 above) having a different origin.

In fact, the work of the present author, pointing out the spurious nature of the LC and RS effects, is doing nothing more than interpreting the space-time LT in the way that it is said in Ref. [2] that it must be —introducing additive constants to correctly describe synchronised clocks at different spatial locations. It is perhaps surprising that Einstein did not do himself what is clearly stated must be done, if clocks at different positions are considered, in the passage quoted in Section 2 above. Even more surprising is that it has also been overlooked, for more than a century, by historians of science and practitioners of SRT

In conclusion, in the present writer’s opinion, the most important message contained in Ref. [2] is the necessity to modify the mechanical theory established by Newton by redefining the Newtonian momentum and energy of any physical object according to Eqns(13.22) and (13.26) above. These formulae were not given explicitly in Ref. [2], but, as shown above, were implicit, in many places, in the results presented in the paper, such as in the TD relation (15.5) leading to the velocity 4-vector of Eqn(15.38), in the transformation laws of the frequency (15.13) and the energy density (15.16) of a plane electromagnetic wave (a parallel beam of monochromatic photons), in the transformation laws of charge densities and currents (15.23)-(15.28) and the mass-energy equivalence relations (15.34)-(15.36). For this alone, and in spite of its many flaws, both of physical principles and in the mathematical derivations presented, Ref. [2] still remains a serious candidate to be considered the most important single physics paper to be published during the 20th Century.

Appendix

§1. Definition of Simultaneity

$$t_B - t_A = t'_A - t_B \quad (\text{E1.1})$$

$$\frac{2AB}{t'_A - t_B} = c \quad (\text{E1.2})$$

§2. On the Relativity of Lengths and Times

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}} = c \quad (\text{E2.1})$$

$$t_B - t_A = \frac{r_{AB}}{c - v} \quad (\text{E2.2})$$

$$t'_A - t_B = \frac{r_{AB}}{c + v} \quad (\text{E2.3})$$

§3. Theory of the Transformation of Co-ordinates and Times from a Stationary System to another System in Uniform Motion of Translation Relative to the Former

$$x' \equiv x - vt = x(t = 0) = \text{constant} \quad (\text{E3.1})$$

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1 \quad (\text{E3.2})$$

$$\frac{1}{2}[\tau(0, 0) + \tau(0, \frac{x'}{c - v} + \frac{x'}{c + v})] = \tau(x', \frac{x'}{c - v}) \quad (\text{E3.3})$$

$$\frac{1}{2} \left(\frac{1}{c - v} + \frac{1}{c + v} \right) \frac{\partial \tau}{\partial t} = \frac{\partial \tau}{\partial x'} + \frac{1}{c - v} \frac{\partial \tau}{\partial t'} \quad (\text{E3.4})$$

$$\frac{\partial \tau}{\partial x'} + \frac{v}{c^2 - v^2} \frac{\partial \tau}{\partial x} = 0 \quad (\text{E3.5})$$

$$\frac{\partial \tau}{\partial y} = 0, \quad \frac{\partial \tau}{\partial z} = 0 \quad (\text{E3.6})$$

$$\tau = a \left(t - \frac{v}{c^2 - v^2} x' \right) \quad (\text{E3.7})$$

$$\xi = c\tau = ac \left(t_B - \frac{vx'_B}{c^2 - v^2} \right) \quad (\text{E3.8})$$

$$t_B = \frac{x'_B}{c - v} \quad (\text{E3.9})$$

]

$$\xi_B = a \frac{c^2}{c^2 - v^2} x'_B \quad (\text{E3.10})$$

$$\eta = c\tau = ac \left(t_B - \frac{vx'_B}{c^2 - v^2} \right) \quad (\text{E3.11})$$

$$\eta = \frac{ac}{\sqrt{c^2 - v^2}} y \quad (\text{E3.12})$$

$$\zeta = \frac{ac}{\sqrt{c^2 - v^2}} z \quad (\text{E3.13})$$

$$\tau = \phi(v)\gamma(t - \frac{vx}{c^2}) \quad (\text{E3.14})$$

$$\xi = \phi(v)\gamma(x - vt) \quad (\text{E3.15})$$

$$\eta = \phi(v)y \quad (\text{E3.16})$$

$$\zeta = \phi(v)z \quad (\text{E3.17})$$

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \quad (\text{E3.18})$$

$$x^2 + y^2 + z^2 = c^2 t^2 \quad (\text{E3.19})$$

$$\xi^2 + \eta^2 + \zeta^2 = c^2 \tau^2 \quad (\text{E3.20})$$

$$t' = \phi(-v)\gamma(-v)(\tau + v\xi/c^2) = \phi(v)\phi(-v)t \quad (\text{E3.21})$$

$$x' = \phi(-v)\gamma(-v)(\xi + v\tau) = \phi(v)\phi(-v)x \quad (\text{E3.22})$$

$$y' = \phi(-v)\eta = \phi(v)\phi(-v)y \quad (\text{E3.23})$$

$$z' = \phi(-v)\zeta = \phi(v)\phi(-v)z \quad (\text{E3.24})$$

$$\phi(v)\phi(-v) = 1 \quad (\text{E3.25})$$

$$\phi(v) = \phi(-v) \quad (\text{E3.26})$$

$$\tau = \gamma(t - \frac{vx}{c^2}) \quad (\text{E3.27})$$

$$\xi = \gamma(x - vt) \quad (\text{E3.28})$$

$$\eta = y \quad (\text{E3.29})$$

$$\zeta = z \quad (\text{E3.30})$$

§4. Physical Meaning of the Equations Obtained In Respect to Moving Rigid Bodies and Moving Clocks

$$\xi^2 + \eta^2 + \zeta^2 = R^2 \quad (\text{E4.1})$$

$$\frac{x^2}{1 - (v/c)^2} + y^2 + z^2 = R^2 \quad (\text{E4.2})$$

$$\tau = \frac{1}{\sqrt{1 - (v/c)^2}} \left(t - \frac{vx}{c^2} \right) \quad (\text{E4.3})$$

$$\tau = t \sqrt{1 - (v/c)^2} = t - (1 - \sqrt{1 - (v/c)^2})t \quad (\text{E4.4})$$

§5. The Composition of Velocities

$$\xi = \omega_\xi \tau, \quad \eta = \omega_\eta \tau, \quad \zeta = 0 \quad (\text{E5.1})$$

$$x = \frac{(\omega_\xi + v)t}{1 + \frac{v\omega_\xi}{c^2}} \quad (\text{E5.2})$$

$$y = \omega_\eta t \frac{\sqrt{1 - (v/c)^2}}{1 + \frac{v\omega_\xi}{c^2}} \quad (\text{E5.3})$$

$$z = 0 \quad (\text{E5.4})$$

$$V^2 = \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \quad (\text{E5.5})$$

$$\omega^2 = \omega_\xi^2 + \omega_\eta^2 \quad (\text{E5.6})$$

$$\alpha = \tan^{-1}(\omega_\eta / \omega_\xi) \quad (\text{E5.7})$$

$$V = \frac{\sqrt{v^2 + \omega^2 + 2v\omega \cos \alpha - [(v\omega \sin \alpha)/c]^2}}{1 + \frac{v\omega \cos \alpha}{c^2}} \quad (\text{E5.8})$$

$$V = \frac{v + \omega}{1 + \frac{v\omega}{c^2}} \quad (\text{E5.9})$$

$$V = c \frac{2c - \kappa - \lambda}{2c - \kappa - \lambda - \kappa\lambda/c} < c \quad (\text{E5.10})$$

$$V = \frac{c + \omega}{1 + \frac{\omega}{c}} = c \quad (\text{E5.11})$$

§6. Transformation of the Maxwell-Hertz Equations for Empty Space. On the Nature of the Electromotive Forces in a Magnetic Field During Motion

$$\frac{1}{c} \frac{\partial E_x}{\partial t} = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}, \quad \frac{1}{c} \frac{\partial B_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \quad (\text{E6.1})$$

$$\frac{1}{c} \frac{\partial E_y}{\partial t} = \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}, \quad \frac{1}{c} \frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \quad (\text{E6.2})$$

$$\frac{1}{c} \frac{\partial E_z}{\partial t} = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}, \quad \frac{1}{c} \frac{\partial B_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \quad (\text{E6.3})$$

$$\frac{1}{c} \frac{\partial E_x}{\partial \tau} = \frac{\partial}{\partial \eta} [\gamma(B_z - \beta E_y)] - \frac{\partial}{\partial \zeta} [\gamma(B_y + \beta E_z)] \quad (\text{E6.4})$$

$$\frac{1}{c} \frac{\partial}{\partial \tau} [\gamma(E_y - \beta B_z)] = \frac{\partial B_x}{\partial \zeta} - \frac{\partial}{\partial \xi} [\gamma(B_z - \beta E_y)] \quad (\text{E6.5})$$

$$\frac{1}{c} \frac{\partial}{\partial \tau} [\gamma(E_z + \beta B_y)] = \frac{\partial}{\partial \zeta} [\gamma(B_y + \beta E_z)] - \frac{\partial B_x}{\partial \eta} \quad (\text{E6.6})$$

$$\frac{1}{c} \frac{\partial B_x}{\partial \tau} = \frac{\partial}{\partial \zeta} [\gamma(E_y - \beta B_z)] - \frac{\partial}{\partial \eta} [\gamma(E_z + \beta B_y)] \quad (\text{E6.7})$$

$$\frac{1}{c} \frac{\partial}{\partial \tau} [\gamma(B_y + \beta E_z)] = \frac{\partial}{\partial \xi} [\gamma(E_z + \beta B_y)] - \frac{\partial E_x}{\partial \zeta} \quad (\text{E6.8})$$

$$\frac{1}{c} \frac{\partial}{\partial \tau} [\gamma(B_z - \beta E_y)] = \frac{\partial E_x}{\partial \eta} - \frac{\partial}{\partial \xi} [\gamma(E_y - \beta B_z)] \quad (\text{E6.9})$$

$$\frac{1}{c} \frac{\partial E'_\xi}{\partial \tau} = \frac{\partial B'_\zeta}{\partial \eta} - \frac{\partial B'_\eta}{\partial \zeta}, \quad \frac{1}{c} \frac{\partial B'_\xi}{\partial \tau} = \frac{\partial E'_\eta}{\partial \zeta} - \frac{\partial E'_\zeta}{\partial \eta} \quad (\text{E6.10})$$

$$\frac{1}{c} \frac{\partial E'_\eta}{\partial \tau} = \frac{\partial B'_\xi}{\partial \zeta} - \frac{\partial B'_\zeta}{\partial \xi}, \quad \frac{1}{c} \frac{\partial B'_\eta}{\partial \tau} = \frac{\partial E'_\zeta}{\partial \xi} - \frac{\partial E'_\xi}{\partial \zeta} \quad (\text{E6.11})$$

$$\frac{1}{c} \frac{\partial E'_\zeta}{\partial \tau} = \frac{\partial B'_\eta}{\partial \xi} - \frac{\partial B'_\xi}{\partial \eta}, \quad \frac{1}{c} \frac{\partial B'_\zeta}{\partial \tau} = \frac{\partial E'_\xi}{\partial \eta} - \frac{\partial E'_\eta}{\partial \xi} \quad (\text{E6.12})$$

$$E'_\xi = \psi(v) E_x, \quad B'_\xi = \psi(v) B_x \quad (\text{E6.13})$$

$$E'_\eta = \psi(v) \gamma[E_y - \beta B_z], \quad B'_\eta = \psi(v) \gamma[B_y + \beta E_z] \quad (\text{E6.14})$$

$$E'_\zeta = \psi(v) \gamma[E_z + \beta B_y], \quad B'_\zeta = \psi(v) \gamma[B_z - \beta E_y] \quad (\text{E6.15})$$

$$\psi(v)\psi(-v) = 1, \quad \psi(v) = \psi(-v), \quad \psi(v) = 1 \quad (\text{E6.16})$$

$$E'_\xi = E_x, \quad B'_\xi = B_x \quad (\text{E6.17})$$

$$E'_\eta = \gamma[E_y - \beta B_z], \quad B'_\eta = \gamma[B_y + \beta E_z] \quad (\text{E6.18})$$

$$E'_\zeta = \gamma[E_z + \beta B_y], \quad B'_\zeta = \gamma[B_z - \beta E_y] \quad (\text{E6.19})$$

§7. Theory of Doppler's Principle and Aberration

$$E_x = E_x^0 \sin \Phi, \quad B_x = B_x^0 \sin \Phi \quad (\text{E7.1})$$

$$E_y = E_y^0 \sin \Phi, \quad B_y = B_y^0 \sin \Phi \quad (\text{E7.2})$$

$$E_z = E_z^0 \sin \Phi, \quad B_z = B_z^0 \sin \Phi \quad (\text{E7.3})$$

$$\Phi = \omega \left\{ t - \frac{1}{c} (lx + my + nz) \right\} \quad (\text{E7.4})$$

$$E'_\xi = E_x^0 \sin \Phi', \quad B'_\xi = B_x^0 \sin \Phi' \quad (\text{E7.5})$$

$$E'_\eta = \gamma[E_y^0 - \beta B_z^0] \sin \Phi', \quad B'_\eta = \gamma[B_y^0 + \beta E_z^0] \sin \Phi' \quad (\text{E7.6})$$

$$E'_\zeta = \gamma[E_z^0 + \beta B_y^0] \sin \Phi', \quad B'_\zeta = \gamma[B_z^0 - \beta E_y^0] \sin \Phi' \quad (\text{E7.7})$$

$$\Phi' = \omega' \left\{ \tau - \frac{1}{c} (l' \xi + m' \eta + n' \zeta) \right\} \quad (\text{E7.8})$$

$$\omega' = \omega \gamma (1 - \beta l) \quad (\text{E7.9})$$

$$l' = \frac{l - \beta}{1 - \beta l} \quad (\text{E7.10})$$

$$m' = \frac{m}{\gamma(1 - \beta l)} \quad (\text{E7.11})$$

$$n' = \frac{n}{\gamma(1 - \beta l)} \quad (\text{E7.12})$$

$$\nu' = \frac{\nu(1 - \beta \cos \phi)}{\sqrt{1 - \beta^2}} \quad (\text{E7.13})$$

$$\nu' = \nu \sqrt{\frac{1 - \beta}{1 + \beta}} \quad (\text{E7.14})$$

$$\cos \phi' = \frac{\cos \phi - \beta}{1 - \beta \cos \phi} \quad (\text{E7.15})$$

$$\cos \phi' = -\beta \quad (\text{E7.16})$$

$$(A')^2 = \frac{A^2(1 - \beta \cos \phi)^2}{1 - \beta^2} \quad (\text{E7.17})$$

$$(A')^2 = \frac{A^2(1 - \beta)}{1 + \beta} \quad (\text{E7.18})$$

§8. Transformation of the Energy of Light Rays. Theory of the Pressure of Radiation Exerted on Perfect Reflectors

$$(x - lct)^2 + (y - mct)^2 + (z - nct)^2 = R^2 \quad (\text{E8.1})$$

$$\xi^2 \frac{(1 - \beta l)^2}{1 - \beta^2} + (\eta - m\gamma\beta\xi)^2 + (\zeta - n\gamma\beta\xi)^2 = R^2 \quad (\text{E8.2})$$

$$\frac{V'}{V} = \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \phi} \quad (\text{E8.3})$$

$$\frac{E'}{E} = \frac{(A')^2 V'}{A^2 V} = \frac{1 - \beta \cos \phi}{\sqrt{1 - \beta^2}} \quad (\text{E8.4})$$

$$\frac{E'}{E} = \sqrt{\frac{1 - \beta}{1 + \beta}} \quad (\text{E8.5})$$

$$A' = A \frac{1 - \beta \cos \phi}{\sqrt{1 - \beta^2}} \quad (\text{E8.6})$$

$$\cos \phi' = \frac{\cos \phi - \beta}{1 - \beta \cos \phi} \quad (\text{E8.7})$$

$$\nu' = \nu \frac{1 - \beta \cos \phi}{\sqrt{1 - \beta^2}} \quad (\text{E8.8})$$

$$A'' = A' \quad (\text{E8.9})$$

$$\cos \phi'' = -\cos \phi' \quad (\text{E8.10})$$

$$\nu'' = \nu' \quad (\text{E8.11})$$

$$A''' = A'' \frac{1 + \beta \cos \phi''}{\sqrt{1 - \beta^2}} = A \frac{(1 - 2\beta \cos \phi + \beta^2)}{1 - \beta^2} \quad (\text{E8.12})$$

$$\cos \phi''' = \frac{\cos \phi'' + \beta}{1 + \beta \cos \phi''} = \frac{2\beta - (1 + \beta^2) \cos \phi}{1 - 2\beta \cos \phi + \beta^2} \quad (\text{E8.13})$$

$$\nu''' = \nu'' \frac{1 + \beta \cos \phi''}{\sqrt{1 - \beta^2}} = \nu \frac{(1 - 2\beta \cos \phi + \beta^2)}{1 - \beta^2} \quad (\text{E8.14})$$

$$f_E = \frac{A^2 (c \cos \phi - v)}{8\pi} \quad (\text{E8.15})$$

$$f_{E'''} = \frac{(A''')^2 (-c \cos \phi''' + v)}{8\pi} \quad (\text{E8.16})$$

$$P = 2 \frac{A^2 (\cos \phi - \beta)^2}{8\pi (1 - \beta^2)} \quad (\text{E8.17})$$

$$P = 2 \frac{A^2}{8\pi} \cos^2 \phi \quad (\text{E8.18})$$

§9. Transformation of the Maxwell-Hertz Equations when Convection-Currents are Taken into Account

$$\frac{1}{c} \left\{ \frac{\partial E_x}{\partial t} + u_x \rho \right\} = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}, \quad \frac{1}{c} \frac{\partial B_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \quad (\text{E9.1})$$

$$\frac{1}{c} \left\{ \frac{\partial E_y}{\partial t} + u_y \rho \right\} = \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}, \quad \frac{1}{c} \frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \quad (\text{E9.2})$$

$$\frac{1}{c} \left\{ \frac{\partial E_z}{\partial t} + u_z \rho \right\} = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}, \quad \frac{1}{c} \frac{\partial B_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \quad (\text{E9.3})$$

$$\rho = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \quad (\text{E9.4})$$

$$\frac{1}{c} \left\{ \frac{\partial E'_\xi}{\partial \tau} + u_\xi \rho' \right\} = \frac{\partial B'_\zeta}{\partial \eta} - \frac{\partial B'_\eta}{\partial \zeta}, \quad \frac{1}{c} \frac{\partial B'_\xi}{\partial \tau} = \frac{\partial E'_\eta}{\partial \zeta} - \frac{\partial E'_\zeta}{\partial \eta} \quad (\text{E9.5})$$

$$\frac{1}{c} \left\{ \frac{\partial E'_\eta}{\partial \tau} + u_\eta \rho' \right\} = \frac{\partial B'_\xi}{\partial \zeta} - \frac{\partial B'_\zeta}{\partial \xi}, \quad \frac{1}{c} \frac{\partial B'_\eta}{\partial \tau} = \frac{\partial E'_\zeta}{\partial \xi} - \frac{\partial E'_\xi}{\partial \zeta} \quad (\text{E9.6})$$

$$\frac{1}{c} \left\{ \frac{\partial E'_\zeta}{\partial \tau} + u_\zeta \rho' \right\} = \frac{\partial B'_\eta}{\partial \xi} - \frac{\partial B'_\xi}{\partial \eta}, \quad \frac{1}{c} \frac{\partial B'_\zeta}{\partial \tau} = \frac{\partial E'_\xi}{\partial \eta} - \frac{\partial E'_\eta}{\partial \xi} \quad (\text{E9.7})$$

$$u'_\xi = \frac{u_x - v}{1 - u_x v / c^2} \quad (\text{E9.8})$$

$$u'_\eta = \frac{u_y}{\gamma(1 - u_x v / c^2)} \quad (\text{E9.9})$$

$$u'_\zeta = \frac{u_z}{\gamma(1 - u_x v / c^2)} \quad (\text{E9.10})$$

$$\rho' = \frac{\partial E'_\xi}{\partial \xi} + \frac{\partial E'_\eta}{\partial \eta} + \frac{\partial E'_\zeta}{\partial \zeta} = \gamma(1 - u_x v / c^2) \rho \quad (\text{E9.11})$$

§10. Dynamics of the Slowly Accelerated Electron

$$m \frac{d^2 x}{dt^2} = e E_x \quad (\text{E10.1})$$

$$m \frac{d^2 y}{dt^2} = e E_y \quad (\text{E10.2})$$

$$m \frac{d^2 z}{dt^2} = e E_z \quad (\text{E10.3})$$

$$m \frac{d^2 \xi}{d\tau^2} = e E'_\xi \quad (\text{E10.4})$$

$$m \frac{d^2 \eta}{d\tau^2} = e E'_\eta \quad (\text{E10.5})$$

$$m \frac{d^2 \zeta}{d\tau^2} = e E'_\zeta \quad (\text{E10.6})$$

$$\xi = \gamma(v - vt), \quad \eta = y, \quad \zeta = z, \quad \tau = \gamma(t - vx/c^2) \quad (\text{E10.7})$$

$$E'_\xi = E_x, \quad E'_\eta = \gamma(E_y - \beta B_z), \quad E'_\zeta = \gamma(E_z + \beta B_y) \quad (\text{E10.8})$$

$$\frac{d^2x}{dt^2} = \frac{e}{m\gamma^3} E_x \quad (\text{E10.9})$$

$$\frac{d^2y}{dt^2} = \frac{e}{m\gamma} (E_y - \beta B_z) \quad (\text{E10.10})$$

$$\frac{d^2z}{dt^2} = \frac{e}{m\gamma} (E_z + \beta B_y) \quad (\text{E10.11})$$

$$m\gamma^3 \frac{d^2x}{dt^2} = eE_x = eE'_\xi \quad (\text{E10.12})$$

$$m\gamma^2 \frac{d^2y}{dt^2} = e\gamma(E_y - \beta B_z) = eE'_\eta \quad (\text{E10.13})$$

$$m\gamma^2 \frac{d^2z}{dt^2} = e\gamma(E_z + \beta B_y) = eE'_\zeta \quad (\text{E10.14})$$

$$\text{Longitudinal mass} = \frac{m}{(\sqrt{1 - \beta^2})^3} \quad (\text{E10.15})$$

$$\text{Transverse mass} = \frac{m}{1 - \beta^2} \quad (\text{E10.16})$$

$$W = \int eE_x dx = m \int_0^v \gamma^3 v dv = mc^2 \left\{ \frac{1}{\sqrt{1 - \beta^2}} - 1 \right\} \quad (\text{E10.17})$$

$$\frac{A_m}{A_e} = \frac{v}{c} \quad (\text{E10.18})$$

$$\mathcal{P} = \int E_x dx = \frac{m}{e} c^2 \left\{ \frac{1}{\sqrt{1 - \beta^2}} - 1 \right\} \quad (\text{E10.19})$$

$$-\frac{d^2y}{dx^2} = \frac{v^2}{R} = \frac{e}{m} \beta B_z \sqrt{1 - \beta^2} \quad (\text{E10.20})$$

$$R = \frac{mc^2}{e} \frac{\beta}{\sqrt{1 - \beta^2}} \frac{1}{B_z} \quad (\text{E10.21})$$

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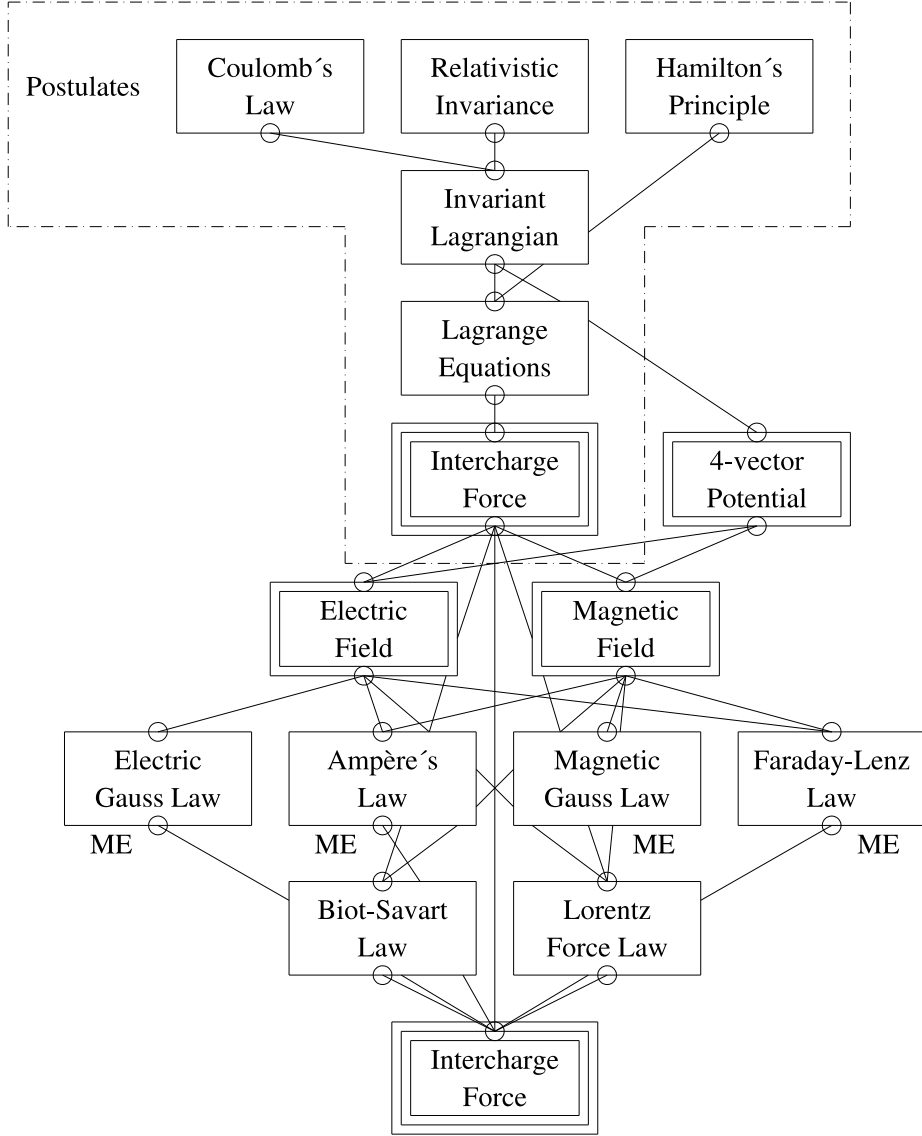


Figure 1: *Concept flow diagram of RCED. The steps in the derivation of the intercharge force equation are inside the dot-dashed line. Downwards-directed lines indicate concept transfer or derivation, e.g. in the case of the 4-vector potential or electric or magnetic fields, by mathematical substitution in a previously derived equation. The ‘Intercharge Force’ is given by Eqn(3.4) below. Maxwell’s Equations, which are all derived, to lowest order in β , in this approach are denoted by ME. The lower part of the figure, outside the dot-dashed line, where the ‘Intercharge Force’ is obtained from Maxwell’s Equations and/or the Biot-Savart and Lorentz Force Laws, corresponds closely with the concept flow of 19th Century CEM shown in Fig.2.*

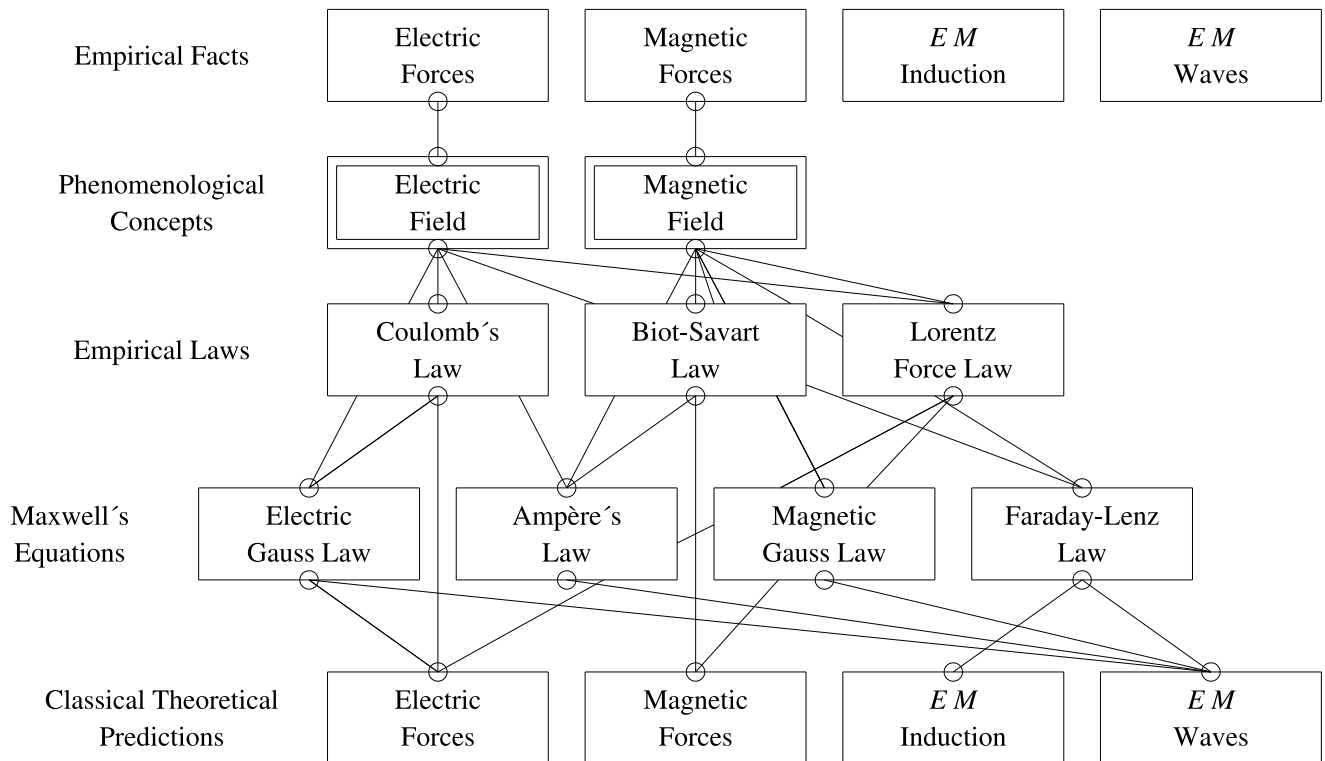


Figure 2: *Concept flow diagram of the 19th Century development of CEM. All 'Empirical Laws' and 'Classical Theoretical Predictions', with the exception of 'EM waves', are included in the single box 'Intercharge Force' of Fig.1.*

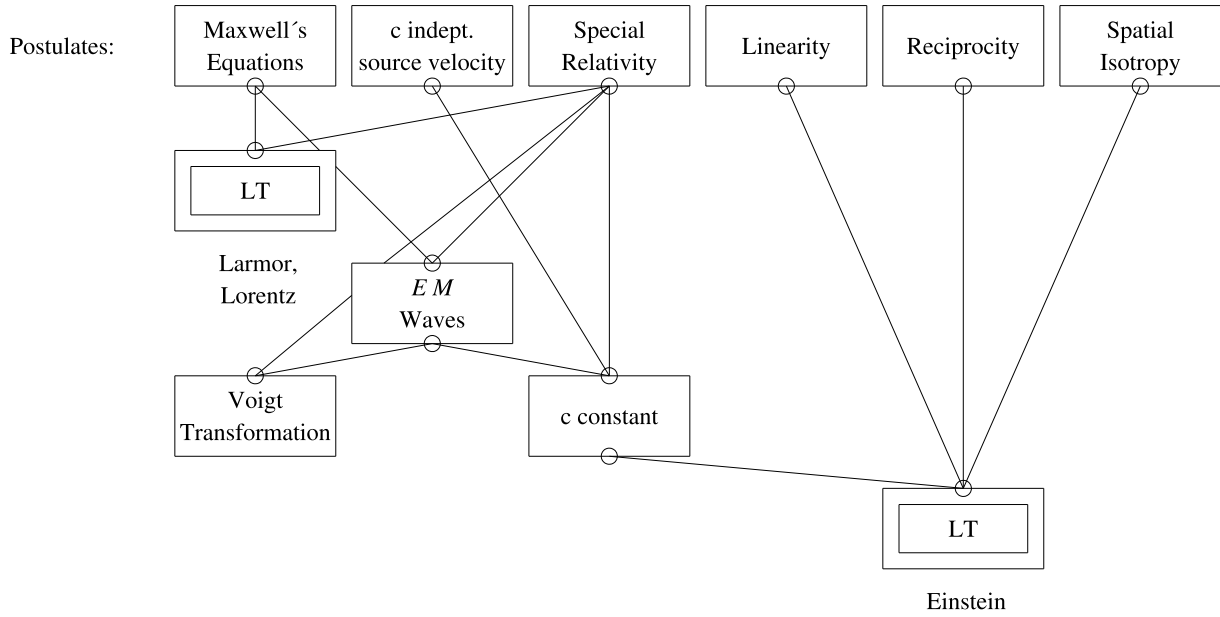


Figure 3: *Concept flow diagram of the derivation of the Voigt Transformation, and of the Larmor-Lorentz and Einstein derivations of the Lorentz Transformation (LT). Einstein's second postulate stated only the source velocity independence of the speed of light. The constancy of the speed of light in all inertial frames was considered by Einstein to be a 'Law of Nature' and to therefore be a direct consequence of the Special Relativity Principle. Additional necessary assumptions for Einstein's derivation are linearity of the equations, the Reciprocity Principle (if the velocity of the inertial frame S' relative to S is \vec{v} , then the velocity of S relative to S' is $-\vec{v}$) and spatial isotropy.*

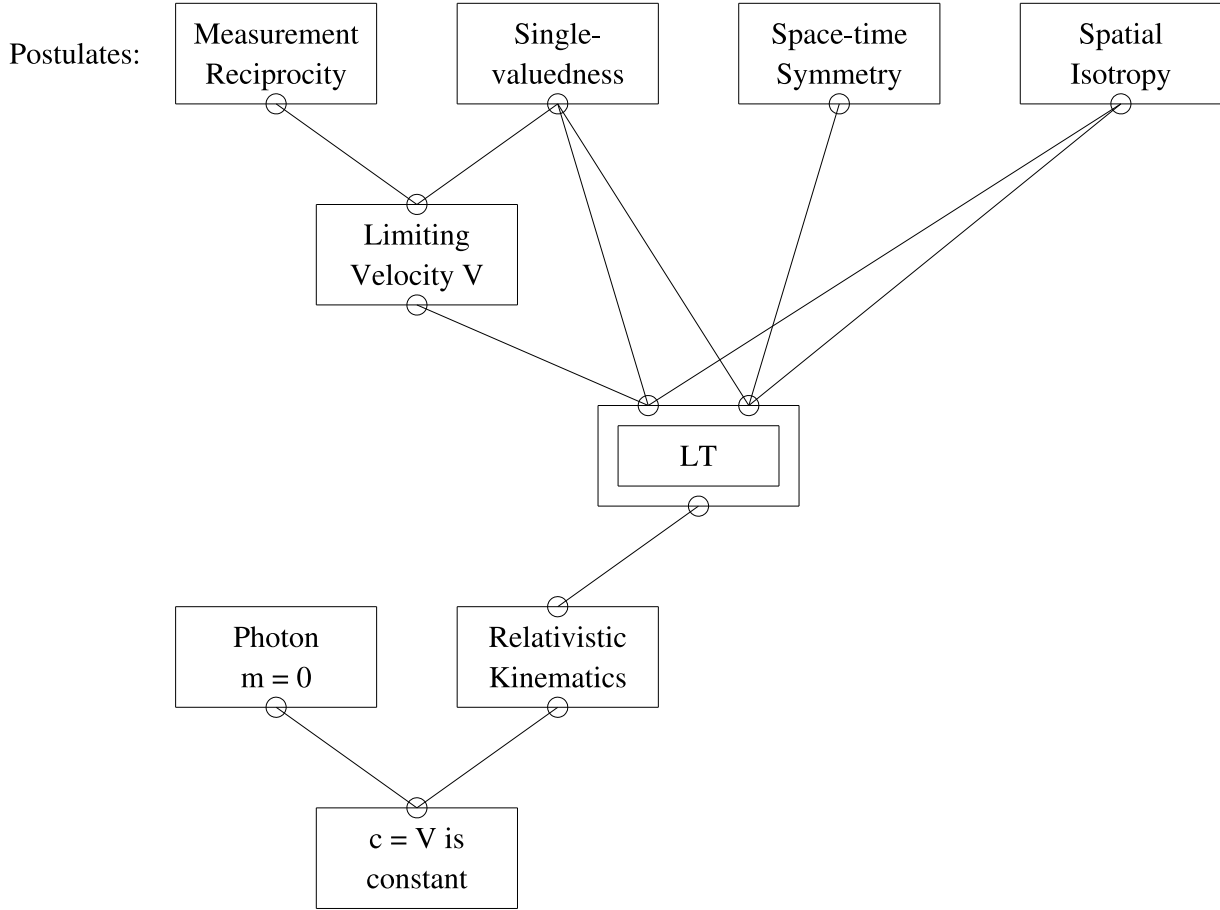


Figure 4: *Concept flow diagram of the derivations of the LT in Refs. [32, 33]. No electro-dynamical concepts or other dynamical laws are invoked. Einstein's second postulate, as well as the constancy of the speed of light, are consequences of relativistic kinematics and the masslessness of the photon. The crucial derivation of the necessary existence of the limiting velocity, V , was already done by Ignatowsky [30] in 1910.*

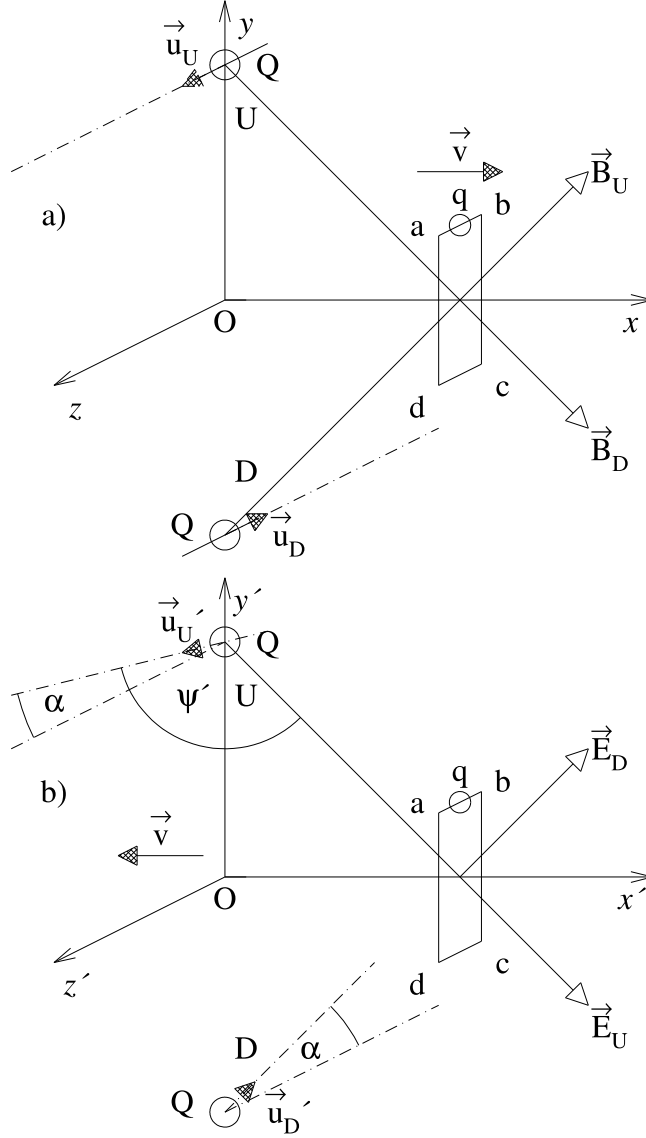


Figure 5: *Electromagnetic induction due to a simple two charge ‘magnet’ in: a) the frame in which the magnet is at rest and the test charge q is in motion and: b) the frame in which the magnet is in motion and the test charge is at rest. In b) the Heaviside formula (3.1) predicts that the electric field of each source charge is radial and so confined to the $x'y'$ plane at the position of the test charge. Since there is then no electrical force in the z' direction the induction effect vanishes. Calculation of the electric force parallel to the z' axis using the RCED formula (3.4) gives a result consistent with the Lorentz force in a) or by use of the Faraday-Lenz Law in either a) or b) [10].*

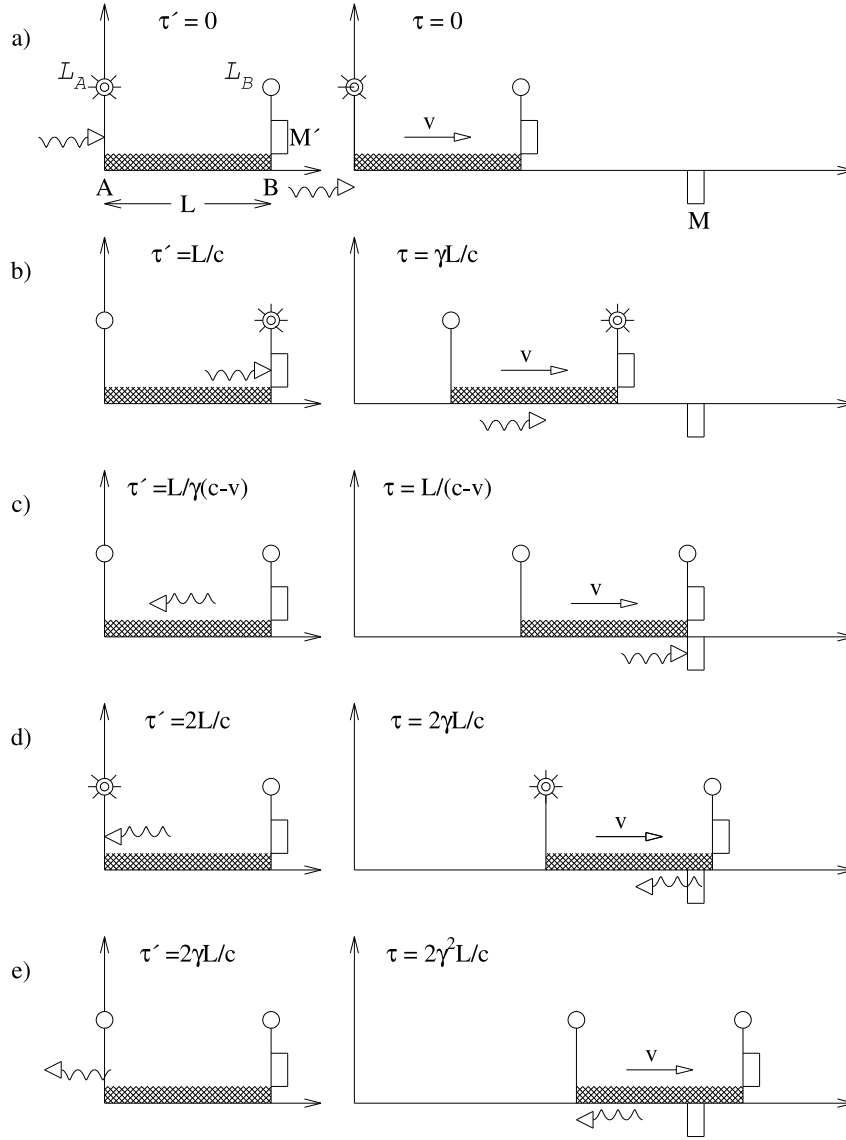


Figure 6: *Einstein's light synchronisation procedure is carried out in the frame S' (left hand figures) and observed in the frame S (right hand figures). The lamps L_A and L_B in S' are triggered by the passage of the light signal in S' and observed from S . a) left, $\tau' = 0$, light signal passes A, L_A fires. b) left $\tau' = L/c$, light signal is reflected at the mirror at B, L_B fires. d) left $\tau' = 2L/c$, light signal arrives back at A, L_A fires a second time. The events observed in S at the corresponding times $\tau = 0$, $\tau = \gamma L/c$, $\tau' = 2\gamma L/c$ are shown in the right-hand figures in a), b), d), respectively. Also shown in a) c) and e) right are the light signal events in the frame S considered by Einstein, together with (left) the positions of the light signal in S' , considered in a), b) and d), at the corresponding times. The position of the light signal is at the head of the wavy arrow in each figure. In this figure $\beta = c/2$, $\gamma = 2/\sqrt{3}$.*

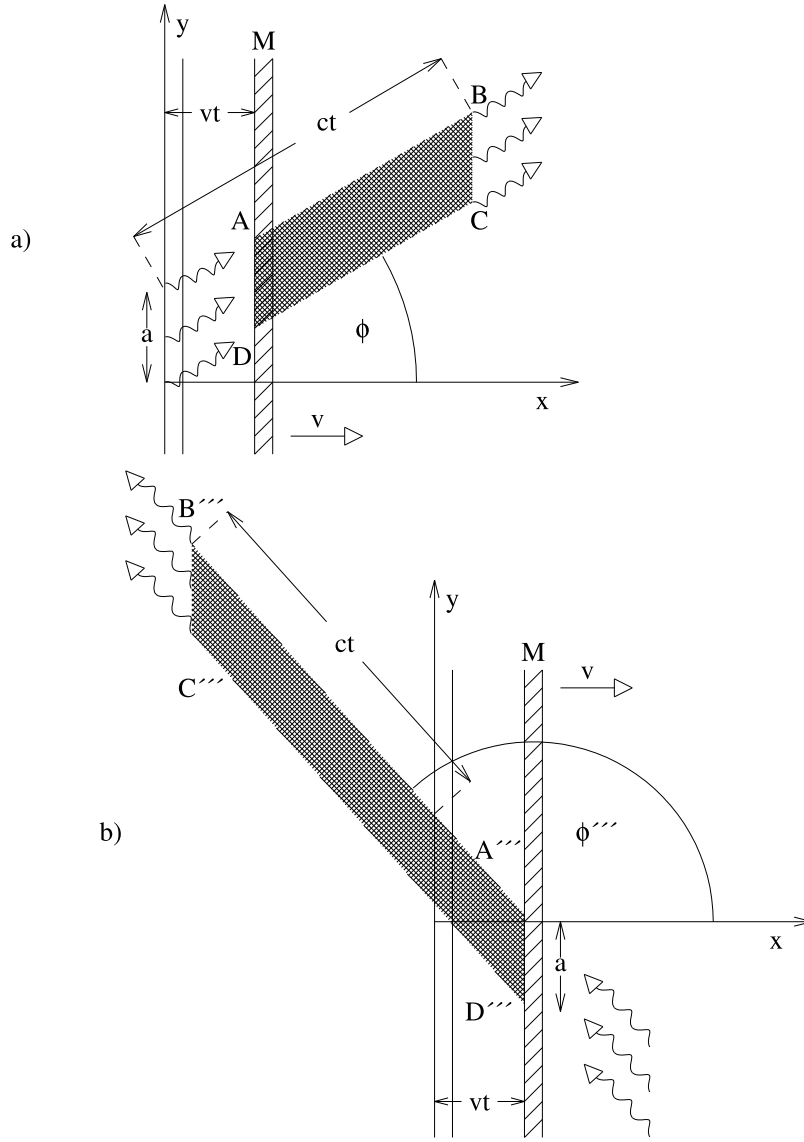


Figure 7: a) a plane electromagnetic wave (a parallel beam of monochromatic photons) in the x - y plane is incident at an angle ϕ on a plane mirror, M , with surface perpendicular to the x -axis moving with uniform velocity v parallel to the latter. The number of photons in the beam of y - and z -dimensions a striking the front surface of the mirror in time t is proportional to the area $ABCD$. b) a parallel beam of monochromatic photons in the x - y plane is incident at an angle $\pi - \phi'''$ on the back surface! of the mirror M . The number of photons in the beam of y - and z -dimensions a striking the back surface of the mirror is proportional to the area $A''B''C''D''$. Einstein incorrectly assumed that this was the number of photons reflected from the front surface of the M in time t . In this figure, $\phi = 30^\circ$, $\phi''' = 132^\circ$ and $v = 0.25c$.